AHP UNDER UNCERTAINTY: A MODIFIED VERSION OF CLOUD DELPHI HIERARCHICAL ANALYSIS

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ABSTRACT

Cloud Delphi Hierarchical Analysis (CDHA) is an Analytic Hierarchical Process (AHP) based method for group decision making under uncertain environments. CDHA adopts appropriate tools for such environments, namely Delphi method, and Cloud model. Adopting such tools makes it a promising AHP variant in handling uncertainty. In spite of CDHA is a promising method, it is still suffering from two main defects. The first one lies in its definition of the consistency index, the second one lies in the technique used in building the pairwise comparisons Cloud models. This paper will discuss these defects, and propose a modified version. To overcome the defects mentioned above, the modified version will depend more on the context of the interval pairwise comparisons matrix while building the corresponding Cloud pairwise comparisons matrix. A simple case study that involves reproducing the relative area sizes of four provinces in Syria will be used to illustrate the modified version and to compare it with the original one.

KEYWORDS

Fuzzy Set Theory Based AHP, Multi-criterion Group Decision Making, Decision Making under Uncertainty, Cloud Model, Delphi Method.

1. INTRODUCTION

Saaty's Analytic Hierarchical Process "AHP" [1],[2],[3] is a theory and methodology for relative measurement [4],[2], which becomes one of the most widely used multiple criteria decision-making tools. AHP has applications in many fields [5] like: social field, political field, engineering, education, industry, government and management …etc. AHP uses a hierarchical model to represent the decision making problem. This hierarchical model can be divided into levels, the topmost level is the 'focus' of the problem, the intermediate levels correspond to criteria and sub-criteria, while the lowest level contains the decision alternatives. The decision maker is asked to provide his pairwise comparison judgment a_{ij} between each two entities A_i , A_j (alternatives or criteria) with the same parent of the hierarchical model. To make comparisons, a scale of numbers should be used to indicate how many times more important or dominant one entity is over another entity with respect to the parent entity. Saaty [2] suggests the use of a 9-points scale to transform the verbal judgments into numerical quantities representing the values of a_{ij}. But there are usually disagreements between the language description and the numeric relation of scale division [6],[7]. One of these disagreements is that the numerical judgments are not well corresponding to the verbal judgments, another disagreement is that, for different persons, the same qualitative verbal judgment represents different meanings, even the same person represents

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different meanings with the same qualitative verbal judgment when he or she is in different situation. Because of these disagreements, we can say that Saaty's qualitative linguistic scale division is not the best way to handle the real world decision making problems. Moreover, a lot of the real world decision making problems are made in uncertain environments. In such environments exact information doesn't exist; therefore, modelling these problems by using crisp numbers isn't the best choice. Many types of uncertainty exist in the real world decision making problems. GEORGE J & Bo [8] have subdivided uncertainty into two categories: Fuzziness and Ambiguity. Another categorization of uncertainty has been identified by Wu & Mendel [9]. According to this categorization, uncertainty has been subdivided into: intra-personal uncertainty which is the uncertainty that a person has about the judgments, and inter-personal uncertainty which is the uncertainty that a group of people have about the judgments, which arises when a group of subjects delivers different judgments. Also Li, et al. [10] mentioned that the randomness is one of the most important uncertainties, "The randomness of concepts means that any concept is not an isolated fact but related to the external world in various ways". Randomness demonstrates that different persons have different perceptions and interpretations of things; the same person can feel differently in different situations.

To overcome the uncertainty limitation of AHP, a lot of uncertainty management approaches have been used for extending AHP. The best known approaches can be classified into three main categories:

1. Interval AHP Approach: this approach depends on interval numbers in representing the pairwise comparison judgments. That means, the decision maker is asked to express his pairwise comparison judgments using intervals of values instead of crisp values. In this approach, the decision maker can express his uncertain judgments easily. The earliest study of this approach was done by Saaty & Vargas [11], and one of its most famous versions has been proposed by Salo and Hämäläinen [12]. [4]

2. D-S Theory Based AHP Approach: this approach incorporates the Dempster-Shafer theory of evidence [13] and AHP. This approach can be divided into the following subapproaches:

- Conventional DS/AHP Approach: this approach allows judgments on groups of decision alternatives to be made. It also offers a measure of uncertainty in the final results. The earliest study of this approach was introduced by Beynon et al. [14],[15], and it has been developed and applied by a number of authors. See [16].

- D-AHP Approach: this approach is a new approach proposed by Deng et al. [17]. This approach extends AHP method by D Numbers. The concept of D numbers extends the Dempster Shafer evidence theory and is more effective in representing uncertain information. In D-AHP, the pairwise comparison are filled by D numbers and all other steps of AHP are extended accordingly.

3. Fuzzy Set Theory Based AHP Approach: this approach is the most common uncertainty management approach used for extending AHP [18]. This approach depends on the fuzzy sets in representing the pairwise comparison judgments. The fuzziness type of uncertainty is best described by fuzzy set theory [19]. Many types of fuzzy sets has been used in this approach. Depending on these types, this approach can be divided into four main subapproaches:

- Conventional Fuzzy AHP Approach: this approach depends on the ordinary (type 1) fuzzy sets in representing the pairwise comparison judgments. In the conventional Fuzzy AHP approach each pairwise comparison judgment is represented as a fuzzy number that is described by a membership function. The membership function denotes the degree to which elements considered belong to the preference set. The earliest study of this approach was done by Van Laarhoven & Pedrycz [20], and one of its most famous versions has been proposed by Chang [21]. See[22],[23].

- Intuitionistic Fuzzy AHP Approach: this approach depends on the Intuitionistic Fuzzy sets. The Intuitionistic Fuzzy sets are defined by using membership functions and nonmembership functions. This type of fuzzy sets has shown definite advantages in handling uncertainty over the ordinary fuzzy sets [23]. The best known versions of the Intuitionistic Fuzzy AHP approach has been proposed by Sadiq & Tesfamariam [18] and by Xu & Liao [23].

- Interval Type 2 Fuzzy AHP Approach: this approach depends on the interval type 2 fuzzy sets in representing the pairwise comparison judgments. Type-2 fuzzy sets generalize type-1 fuzzy sets so that more uncertainty for defining membership functions can be handled. An interval type-2 fuzzy set is a special case of a type-2 fuzzy set. Interval type-2 fuzzy set can be described in terms of an upper membership function and a lower membership function. Each pairwise comparison judgment is represented as an interval type 2 fuzzy set. The earliest studies of this approach was done by Kahraman, et al. [24] and by Abdullah & Najib [25].

- Cloud AHP Approach: this approach depends on Cloud model in representing the pairwise comparison judgments. Cloud model is a generalization of the ordinary(type 1) fuzzy sets. This model, as a fuzzy set model, is effective in handling the fuzziness type of uncertainty. In addition, it is effective in handling the randomness [10]. In Cloud AHP approach, the decision maker is asked to express his judgments using interval numbers, then the interval pairwise comparison matrices are converted into corresponding Cloud matrices. The earliest study of this approach was done by Yang, et al. [6]. In this study a new AHP variant for handling individual decision making problem under uncertainty has been proposed. This variant is called Cloud Hierarchical Analysis. Later on, Yang., Yang, et al.[7] proposed a new method to handle group decision making problems under uncertainty by integrating the Cloud Hierarchical Analysis, the Delphi method and an effective group decision making technique. This method is called Cloud Delphi Hierarchical Analysis (CDHA).

CDHA strength is its ability to handle both intra-personal and inter-personal uncertainties. In spite that strength, CDHA is still suffering from two fundamental defects. The first defect lies in its definition of the consistency index which is not always true. The second defect lies in the technique used in building the pairwise comparisons Cloud models from the interval pairwise comparisons, which ignores the interval pairwise comparison judgments' randomness, and extracts the cloud pairwise comparisons in an inaccurate manner.

The (section 2) of this paper, explains these defects after listing the CDHA main steps and explaining its basic concepts. A new modified version of CDHA avoiding these defect is presented in (section 3). In (section 4), a simple case study that involves reproducing the relative area sizes of four provinces in Syria is used to compare the original CDHA with the new modified version. Finally, conclusions and suggestions for future researches are presented in (section 5).

2. CLOUD DELPHI HIERARCHICAL ANALYSIS (CDHA)

CDHA [7] is multi-criteria group decision-making method under uncertain environments. It is a modified version of the Cloud Hierarchical Analysis [6], which is upgraded to be a multicriteria group decision-making method. CDHA is based on the following concepts : AHP, normal Cloud model, and the Delphi feedback method.

2.1. CLOUD MODEL

The Cloud model, proposed by Li et al. [26],[27],[10], is a cognitive model which can synthetically describe the randomness and fuzziness of concepts. This model was developed based on the ordinary (type-1) fuzzy set. The aim of Cloud model is to deal with the uncertainty of the membership function which is not considered in ordinary fuzzy sets. According to the ordinary fuzzy set theory, once the membership function is determined, one and only one accurate membership degree can be calculated for any given element in the universe, to measure the uncertainty of this element belonging to the associated concept. This is obviously inconsistent with the spirit of the fuzzy set, because the uncertainty of an element belonging to a fuzzy concept becomes certain and precise in this case. Thus, Li et al. Defined Cloud model by considering whether allowing a stochastic disturbance of the membership degree encircling a determined central value is more feasible [7].

2.2. NORMAL CLOUD MODEL

Among several kinds of defined Cloud models, the normal Cloud model based on normal distribution and Gaussian membership function is the most commonly used. In this paper, we discuss only the normal Cloud model, and the name of Cloud model is regarded as equivalent to the normal Cloud model. The normal Cloud model can effectively integrate the randomness and the fuzziness of concepts. It describe the overall quantitative property of a concept by three numerical characteristics: Expectation Ex, Entropy En, and Hyperentropy He. Ex is the mathematical expectation of the cloud drops belonging to a concept in the universe. It can be regarded as the most representative and typical sample of the qualitative concept. En represents the uncertainty measurement of a qualitative concept. It is determined by both the randomness and the fuzziness of the concept. In one aspect, as the measurement of randomness, En reflects the dispersing extent of the cloud drops. In the other aspect, it is also the measurement of fuzziness, representing the scope of the universe that can be accepted by the concept. He is the uncertain degree of entropy En, also seen as the entropy of entropy (the uncertainty of uncertainty). He reflects the dispersion of the Cloud drops. As He is getting bigger, the Cloud's dispersion, thickness and the randomness of degree of membership are getting bigger. An example of a normal Cloud with the following parameters (Ex=0.645,En=0.042 and He=0.008) is presented in Figure 1. The normal Cloud model is defined as follows :

Definition 1 [10]: Let U be the universe of discourse and $\widetilde{A}$ be a qualitative concept in U. If $x \in U$ is a random instantiation of concept $\widetilde{A}$, which satisfies $x \sim N(Ex, \grave{En}^2)$, $\grave{En} \sim N(En, He^2)$, and the certainty degree of x belonging to concept $\widetilde{A}$ satisfies

$$\mu(x) = e^{-\frac{(x-Ex)^2}{2(\grave{En})^2}}$$

then the distribution of x in the universe U is called as a normal Cloud.

Notice: Arithmetic operations have been defined on the normal cloud models and have been applied in CDHA [27],[7].

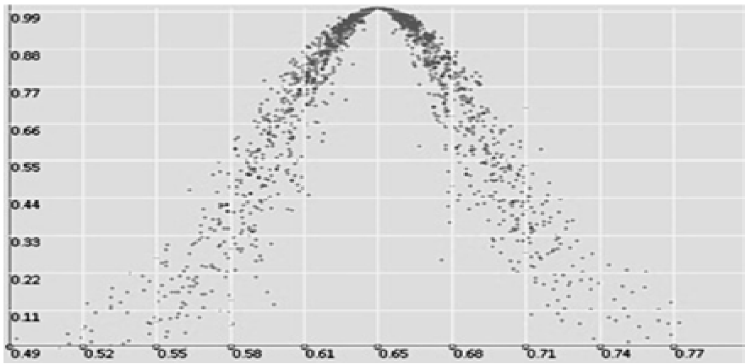


Figure 1. Normal Cloud Model (Ex=0.645, En=0.042, He=0.008)

2.3. DELPHI METHOD

Delphi method [28],[29] is an iterative process used to collect and distil the judgments of experts using several rounds of anonymous written questionnaire interspersed with feedback. The questionnaires are designed to focus on problems, opportunities, solutions, or forecasts. Each subsequent questionnaire is developed based on the results of the previous questionnaire. Delphi method establishes an effective group communication process by providing feedback about information contributions and an assessment of group judgments to enable individuals to re-evaluate their judgments. Delphi method has three features: anonymous response, iteration and controlled feedback, and statistical group response. Since its development in the 1960s at Rand Corporation, Delphi method has been widely accepted in many industry sectors including health care, business, education, information technology, transportation and engineering ..etc.

2.4. CDHA MAIN STEPS

The main steps of CDHA are [7]:

- The initial step which defines the hierarchy of the evaluation, the criteria's weights and the decision makers in addition to their initial importance weights and the factor of updating these weights.
- The first step gathers the individual interval pairwise comparison matrices.
- The second step converts the individual interval pairwise comparisons matrices to Cloud comparison matrices, defines the consistency index (CI) for each comparison matrix and updates the decision makers' weights depending on these consistency indices.
- The third step calculates the synthetic Cloud matrix and the weighted average Cloud matrix. and returns them graphically to the decision makers while applying one-iteration Delphi method.
- The fourth step calculates the individual Cloud weight vectors from the Cloud matrices, and obtains the final group Cloud weight vector by using the weighted geometric mean method.

- Finally, CDHA discusses the ranking of alternatives.

This paper will only explain the second step, because it is responsible for the CDHA main defects.

2.5. Building the Cloud comparison matrices and calculating consistency indexes

In CDHA, we derive the Cloud matrix from the interval pair wise comparison matrix
$A=[a_{ij}]$, i,j = 1, 2, . . . , n, where aij = $[a_{ij}^{Lower}, a_{ij}^{uppe}$r], $a_{ij} = 1/a_{ij}$, and $a_{ij} = 1$

a_{ij}^{lower}, and a_{ij}^{upper} are the lower and upper bounds of the interval. For a given interval pair wise comparison matrix element aij, we define the corresponding Cloud ratio parameters as follows:

$$Ex_{ij} = \frac{a_{ij}^{upper} + a_{ij}^{lower}}{2}, \quad En_{ij} = \frac{a_{ij}^{upper} - a_{ij}^{lower}}{6}, \quad He_{ij} = \frac{\mathrm{Max}\left|En_{ijk} - En_{ij}\right|}{3}$$

$$\text{Where } En_{ijk} = \frac{a_{ijk}^{upper} - a_{ijk}^{lower}}{6}$$

$$\text{and } a_{ijk} = \left[a_{ijk}^{lower}, a_{ijk}^{upper}\right] = \left[a_{ik}^{lower} a_{kj}^{lower}, a_{ik}^{upper} a_{kj}^{upper}\right] \quad k = 1,2.., n, k \neq i, k \neq j$$

Ex expresses the expectation of the interval judgment. Thus, the use of the median of the interval is natural. Interval [Ex -3En, Ex +3En] best represents the qualitative judgment (99.74%, 3En rule). Therefore, 6En can be used to reflect the bound and fuzziness of the interval number. The computation of the uncertainty parameter He is based on the context of the judgment matrix. He also reflects the consistency of the judgment matrix. Thus, CDHA defines the consistency index CI of the Cloud matrix as follows:

$$CI = \frac{1}{n(n-1)} \sum_{\substack{i,j=1 \\ i \neq j}}^{n} \left(He_{ij} / Ex_{ij}\right)$$

2.6. CDHA Defects

The two main defects of CDHA are:
1. The first defect: CDHA definition of the comparison matrix consistency index is incomplete: In CDHA, the consistency index is a very important variable because it is used in updating the decision makers' importance weights which are used in aggregating their opinions. In its definition of the consistency index, CDHA depends on the following proposition: "The parameter He reflects the consistency of the judgment matrix". But, this proposition is not always true. To prove that, we will use the following counter-example: Let us suppose that we have the following inconsistent decision maker's interval pairwise comparison matrix which represents the interval pairwise comparisons between the alternatives A_i , 0<i<4 :

$$A = \begin{array}{c c} & \begin{array}{ccc} A1 & A2 & A3 \end{array} \\ \begin{array}{c} A1 \\ A2 \\ A3 \end{array} & \begin{pmatrix} 1 & [-,-] & [5,5] \\ [5,5] & 1 & [-,-] \\ [-,-] & [5,5] & 1 \end{pmatrix} \end{array}$$

According to the CDHA cloud model building technique, A will be converted into the following cloud comparison matrix Ȧ:

$$\dot{A} = \begin{array}{c} \\ A1 \\ A2 \\ A3 \end{array} \begin{array}{c} \begin{array}{ccc} A1 & A2 & A3 \end{array} \\ \begin{pmatrix} (Ex = 1, En = 0, He = 0) & (Ex = \frac{1}{5}, En = 0, He = 0) & (Ex = 5, En = 0, He = 0) \\ (Ex = 5, En = 0, He = 0) & (Ex = 1, En = 0, He = 0) & (Ex = \frac{1}{5}, En = 0, He = 0) \\ (Ex = \frac{1}{5}, En = 0, He = 0) & (Ex = 5, En = 0, He = 0) & (Ex = 1, En = 0, He = 0) \end{pmatrix} \end{array}$$

Both A and Ȧ are inconsistent because A2 is preferred to A1 and A3 is preferred to A2, but A1 is preferred to A3. Despite that, According to CDHA, the consistency index of Ȧ is CI=0, which means that Ȧ consistency is optimal. Moreover, the parameter He is equal to "0" for each cloud element of Ȧ , which means, in this counter example, the proposition that says He reflects the consistency of the judgment matrix is wrong.

2. The second defect: the technique used in building the pair wise comparisons Cloud models from the interval pair wise comparisons ignores the interval pair wise comparison judgments randomness and extracts the cloud pair wise comparisons in an inaccurate manner. CDHA depends only on a single interval pair wise comparison judgment aij= a_{ij}^{upper}, a_{ij}^{lower} in calculating both Ex_{ij} and En_{ij}. But, this single interval pairwise comparison judgment is affected by the decision maker's mood changes during providing a large number of interval pairwise comparisons. In such situation, the decision maker's mood can be affected by many factors, like boredom, lack of attention and lack of interest. For example, if we ask the decision maker about his degree of preference of alternative Ai to alternative A_j at the beginning of the evaluation process, he may say a_{ij} = [2,3], but if we ask him the same question after providing twenty interval pairwise comparison judgments, he may provide another interval, maybe a_{ij} = [1.5,2]. Depending on the previous discussion, a_{ij} is a single interval measurement of the decision maker's degree of preference of A_i to A_j. a_{ij} contains randomness, and can be considered as a sample of an interval-valued random variable. This interval-valued random variable is defined as follows:

> **Definition 2 :** $R_{ij} = [R_{ij}^{lower}, R_{ij}^{upper}]$ is an interval-valued random variable, its values are the measurements of the decision maker's degree of preference of A_i to A_j. Every measurement can be taken by asking the decision maker about his degree of preference of A_i to A_j.

En_{ij} , According to its definition, represents the uncertainty measurement of the decision maker's degree of preference of Ai to A_j , and it is determined by both the randomness and the fuzziness of the decision maker's degree of preference of A_i to A_j. That means En_{ij} must not ignore the randomness of R_{ij} . But, the calculation formula of Enij in CDHA ignores this randomness, because it depends only on a single sample of R_{ij} . Therefore, the calculation formula of E_{ij} is inaccurate. By following the same logic, the calculation formula of Ex_{ij} is inaccurate; i.e. the expectation of the decision maker's degree of preference of Ai to A_j cannot be determined accurately, if the randomness of R_{ij} is ignored.

Such inaccurate parameters lead to inaccurate thick, filled or even fuzzily-useless pairwise comparisons Cloud models, which have a deep impact on the Cloud weight vector (the output of CDHA). The Cloud weight vector may contain filled or fuzzily useless Clouds because of that impact, and become less useful.

Definition 3 : A filled Cloud model is a Cloud model that verifies the following condition $En_{ij} > He_{ij} \geq 1/3En_{ij}$. While a fuzzily-useless Cloud model is a Cloud model that verifies the following condition $He_{ij} \geq En_{ij}$.

On the one hand, the probability theory considers that fuzzily-useless Cloud models are useful, but, on the other hand, the fuzzy set theory finds them useless. For example, if we considered the fuzzily-useless Cloud model presented in (Figure 2) as a Cloud weight vector element that outlines the relative importance of an alternative Ai, we can extract the following two pieces of information from this Cloud model, the first one is that the expected relative importance of Ai is 0.45, and this information is useful, while the other one is that the membership degree of 0.45 to the relative importance of Ai could be any value between 0 and 1, which is useless.

According to the previous discussion, the filled Cloud models are the same as the fuzzilyuseless Cloud models, but actually, they are not the same. That is because a filled Cloud model can be approximated by an acceptable interval-type 2 fuzzy membership function while a fuzzily-useless Cloud models cannot be approximated by such membership function. Let us suppose that we have a Cloud model C. The thinnest interval-type 2 fuzzy membership function that can be accepted as approximation to the Cloud model C according to the normal distribution's 68% rule is the following membership function f:

$$f^{upper}(x) = e^{-\frac{(x-Ex)^2}{2(En+He)^2}}, f^{lower}(x) = e^{-\frac{(x-Ex)^2}{2(En-H\ )^2}}.$$

The previous function f cannot be used to approximate C if it is a fuzzily-useless Cloud model, because (En-He) will be ≤0, but, it can be used to approximate C if it is a filled Cloud model, because (En-He) will be >0. That is why the fuzzy set theory sees the filled Cloud models more useful than fuzzily-useless Cloud models.

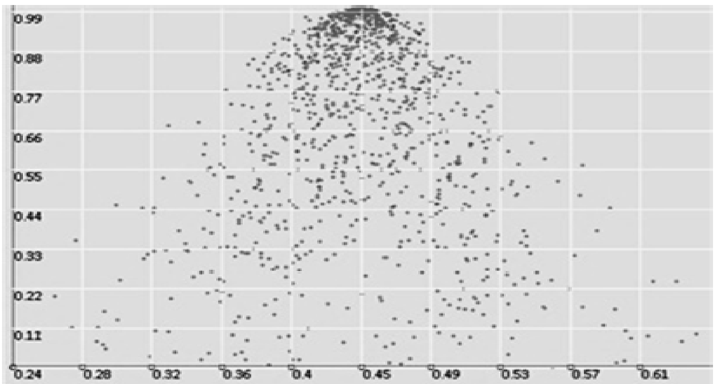


Figure 2. Fuzzily-useless Cloud Model(Ex=0.45, En=0.032, He=0.036)

3. A MODIFIED VERSION OF CDHA

In this section, we will propose a modified version of CDHA. This version will only modify the second step of CDHA (section 2.4). Before getting into the modified version, let us introduce the following definitions:

Let A be an interval pair wise comparison matrix. Each element $a_{ij} = [a_{ij}^{Lower}, a_{ij}^{Upper}]$ of A is an interval that represents the decision maker's degree of preference of alternative A_i to alternative A_j:

$$A = \begin{pmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{n1} & \cdots & a_{nn} \end{pmatrix}$$

We can define the following set:

$$I_{ij} = \{\left[a_{ik}^{lower} a_{kj}^{lower}, a_{ik}^{upper} a_{kj}^{upper}\right] : 1 \leq i, j, k \leq n\}$$

The previous set I_{ij} represents all possible interval values of the decision maker's degree of preference of the alternative Ai to the alternative A_j according to the context of the matrix A. In addition to the previous definition of the set I_{ij}, we define the following two sets:

$$I_{ij}^{upper} = \{a_{ik}^{upper} a_{kj}^{upper} : 1 \leq i, j, k \leq n\}$$

$$I_{ij}^{lower} = \{a_{ik}^{lower} a_{kj}^{lower} : 1 \leq i, j, k \leq n\}$$

The previous sets represent the sets of the upper and lower bounds of the interval elements of the set I_{ij}. Depending on the previous definitions, we can propose the following definition:

$$a_{ij}^{max} = [\min(I_{ij}^{lower}), \max(I_{ij}^{upper})]$$

The upper and lower bounds of the previous interval a_{ij}^{max} represent the maximum and minimum possible values of the decision maker's degree of preference of the alternative A_i to A_j according to the context of the matrix A. It means that this interval includes all possible values of this degree of preference according to the context of the matrix A.

3.1. Consistency Index

Our goal in this section is defining a valid consistency index of the interval pairwise comparison matrix A, To do that, we will convert A into a Cloud matrix called $A^{consistency}$. The matrix $A^{consistency}$ will be used only in calculating the consistency index, and will never be used in any other step of our modified version of CDHA. $A^{consistency}$ is defined as follows:

Let $a_{ij}^{consistency}$ be a Cloud element of the Cloud matrix $A^{consistency}$. The Cloud parameters of $a_{ij}^{consistency}$ will be as follows:

$$Ex_{ij}^{consistency} = \frac{a_{ij}^{lower} + a_{ij}^{upper}}{2}$$

$$En_{ij}^{consistency} = \frac{a_{ij}^{upper} - a_{ij}^{lower}}{6}$$

$$He_{ij}^{consistency} = \frac{En_{ij}^{max} - En_{ij}^{consistency}}{3}, \text{where } En_{ij}^{max} = \frac{\max(I_{ij}^{upper}) - \min(I_{ij}^{lower})}{6}$$

From the previous equations, we can see that both $Ex_{ij}^{consistency}$ and $En_{ij}^{consistency}$ are calculated by using the same calculation formulas which are used in the original CDHA, while the calculation formula of $He_{ij}^{consistency}$ is modified. According to this calculation formula, the parameter $He_{ij}^{consistency}$ reflects the discordance degree between the interval elements of the set I_{ij} and the interval value aij. Depending on $He_{ij}^{consistency}$, we can define the following modified consistency index:

$$\acute{C}I = \frac{1}{n(n-1)} \sum_{\substack{ij=1 \\ i \neq j}}^{n} (He_{ij}^{consistency} / Ex_{ij}^{consistency})$$

The smaller the CI, the better the non-discordance of the pair wise comparisons. In practice, C`I satisfying C`I < 0.1 is required. That is, the average randomness (uncertainty) should be smaller than 10% of the expectation. The modified consistency index of the inconsistent matrix A (section "CDHA Defects") is C`I = 3.472 while the original consistency index of A is CI=0, which means that C`I can avoid the limitations of the original CI.

3.2. BUILDING THE CLOUD MODELS

Our goal in this section is building easy to calculate and more rational Cloud models from the interval pair wise comparisons. Any rational Cloud model of the decision maker's degree of preference of A_i to A_j must have rational parameters. The most rational value of Ex_{ij} is the expected value of the decision maker's degree of preference of A_i to A_j. The previous value is denoted by the following formula:

$C_{ij} = Ex\left(CP(R_{ij})\right)$ where Ex() is the expectation, $CP(R_{ij})$ denotes the central point of the interval R_{ij}.

Enij represents a measurement of the expected uncertainty of the decision maker's degree of preference of A_i to A_j. Which means that, a rational value of En_{ij} could be mined from the following value (L_{ij}) which represent the expected interval length of the interval judgment:

$L_{ij} = Ex\left(L(R_{ij})\right)$ where Ex() is the expectation, $L(R_{ij})$ denotes the interval length of the interval R_{ij}.

According to the previous discussion, the following interval could be one of the best choices to extract both Ex_{ij} and En_{ij}.

$G_{ij} = [C_{ij} - L_{ij}/2\,,\ C_{ij} + L_{ij}/2] = [Ex(R_{ij}^{lower}), Ex(R_{ij}^{upper})]$, where Ex()is the expectation.

In this section, we try to find an approximation of G_{ij}, then we calculate Ex_{ij} and En_{ij} depending on this approximation, after that we calculate He_{ij} depending on En_{ij}. A possible approximation of G_{ij} could be extracted from I_{ij}.That is because, each element of Iij is either a sample of Rij or an approximated sample of R_{ij}. This approximation of G_{ij} can be defined as follows:

$$G_{ij} \approx \grave{a}_{ij} = [\grave{a}_{ij}^{lower}, \grave{a}_{ij}^{upper}]$$

$$\grave{a}_{ij}^{lower} = (1-\beta)\, a_{ij}^{lower} + \beta\, \text{average}(\{I_{ij}^{lower} \backslash \{a_{ii}^{lower} a_{ij}^{lower}, a_{ij}^{lower} a_{jj}^{lower}\}\})$$

$$\grave{a}_{ij}^{upper} = (1-\beta)\, a_{ij}^{upper} + \beta\, \text{average}(\{I_{ij}^{upper} \backslash \{a_{ii}^{upper} a_{ij}^{upper}, a_{ij}^{upper} a_{jj}^{upper}\}\})$$

Where $0 < \beta < 1$, and $a_{ii}^{lower} = a_{jj}^{lower} = a_{ii}^{upper} = a_{jj}^{upper} = 1$.

The elements of $\{I_{ij} \backslash \{a_{ij}\}\}$ are not samples of R_{ij}, but they can be considered as approximated samples of it. β represents our confidence degree of considering the elements of $\{I_{ij} \backslash \{a_{ij}\}\}$ as approximated samples of R_{ij}.The larger β is, the larger our confidence degree of considering the elements of $\{Iij \backslash \{a_{ij}\}\}$ as approximated samples of R_{ij} is. If $(1-\beta) = 2/|I_{ij}|$, where $|I_{ij}|$ is the number of element of I_{ij}, then our confidence degree of considering the elements of $\{I_{ij} \backslash \{a_{ij}\}\}$ as approximated samples is very high.

$\grave{a}_{ij}$ could be a better choice for extracting both Ex_{ij} and En_{ij} than a_{ij}. That is because $\grave{a}_{ij}$ is extracted from more than one approximated sample of Rij, and it does not ignore its randomness. To calculate both Ex_{ij} and En_{ij} from $\grave{a}_{ij}$ the following formula can be used:

$$Ex_{ij} = \frac{\grave{a}_{ij}^{lower} + \grave{a}_{ij}^{upper}}{2}, \qquad En_{ij} = \frac{\grave{a}_{ij}^{upper} - \grave{a}_{ij}^{lower}}{6}$$

A rational value of He_{ij}, which is the entropy of entropy, can be mined from the difference between En_{ij} and the entropy of the longest possible interval according to the interval pairwise comparison matrix context. The calculation formula of the previous value will be:

$$He_{ij} = \frac{En_{ij}^{max} - En_{ij}}{3}, \quad \text{where } En_{ij}^{max} = \frac{\max(I_{ij}^{upper}) - \text{mi}\ (I_{ij}^{lower})}{6}$$

4. Experimental Study

4.1. Reproducing The Relative Area Sizes Of Four Provinces In Syria

This section presents a simple case study designed to verify and illustrate our modified version of CDHA. This case study involves reproducing the relative area sizes of four provinces in Syria. Syria map has four provinces labeled as follows: 1, Damascus; 2, Homs; 3, Sweida'a; 4, Dara'a (Figure 3). This case study is used to compare the final results of our modified version of CDHA with the final results of the original CDHA (section 4.2).

Figure 3. Map Of Syria

A software tool that implements the modified version of CDHA has been developed. Three decision makers, of different professions, named; Etab, Oula and Suleiman, were invited to participate in the process. These decision makers respectively are civil engineer, medicine, and lawyer.

At the beginning, we assigned the parameters of our decision making problem to the software. These parameters are:

- The alternatives: the four provinces.
- The criterion: the province's area.
- The decision makers: Etab, Oula, and Suleiman.
- The Initial weight of each decision makers: 0.333 (equal weights)
- The factor of updating the decision makers' weights [7]: α=0.5 (the importance of the initial weight equals the importance of the weight calculated depending on the comparison matrix consistency index)
- $\beta=1-2/|I_{ij}|=1-2/4=1/2$.

Next, the software generated a computerized questionnaire. This questionnaire contains an empty 4x4 interval pair wise comparison matrix (Table 1).

	Damascus	Homs	Sweida'a	Dara'a
Damascus	[1,1]	[,]	[,]	[,]
Homs	[,]	[1,1]	[,]	[,]
Sweida'a	[,]	[,]	[1,1]	[,]
Dara'a	[,]	[,]	[,]	[1,1]

Table 1. The first computerized questionnaire

Each decision maker filled in his copy of the computerized questionnaire using Figure 4. After that, the software collected the computerized questionnaires and computed both the synthetic and the weighted average Cloud matrices, then it returned the Cloud graphics to the decision makers to re-evaluate the ratios. The whole process is described in detail as follows:

1. The first step is: gathering the individual interval pair wise comparison matrices ; these individual interval pair wise comparison matrices are presented in Table 2.In this step, the software provides each decision maker with an empty interval pair wise comparison matrix (Table 1). The decision maker fills the cells a_{ij} with values larger than 1. Then the software automatically fills the cells in the symmetrical positions using the following formula $a_{ij} = [1/a_{ij}^{Upper}, 1/a_{ij}^{lower}]$.
2. The second step is: calculating the consistency index (C`I) for each interval pair wise comparisons matrix as seen in Table 2, then updating the decision makers weights (Table 2) and finally converting the individual interval pair wise comparisons matrices into Cloud comparison matrices (Table 3). As shown in Table 2 all the consistency indices of the comparison matrices are less than 0.1, thus all of these matrices are consistent and all of these decision makers' comparison matrices are accepted. Also we can see that there is no big difference among these consistency indices, because the problem of reproducing the relative area sizes of our provinces is a non-ambiguous problem. According to these consistency indices, the decision makers' initial weights will not change very much after being updated as shown in Table 2. From Table 2 and Table 3 we can notice that the elements of the Cloud pair wise comparisons matrices do not match their corresponding interval pair wise comparison matrices elements. For example, the cloud pair wise comparison between Homs and Dara'a, according to Suleiman's cloud pairwise comparison

matrix is (Ex=11.5962, En=0.1413, He=0.0879) while its corresponding interval pairwise comparison between Homs and Dara'a is [11,11]. This difference is due to the fact that the cloud pair wise comparison are extracted in a rational manner depending on the whole context of the interval pair wise comparison matrix.

3. The third step: is calculating the synthetic Cloud matrix and the weighted average Cloud matrix as shown in Table 4, then presenting them graphically to the decision makers as feedback information while applying the one-iteration Delphi method. The feedback information sums up the results of the first phase of the Delphi method. It is used to help the decision makers to distil their judgments. A sample of the graphical representation of these feedback information is presented in Figure 4. This figure represents the feedback information, which are introduced to each decision maker about the pair wise comparison between (2:Homs) and (3:Sweida'a). In this figure we can see that the feedback information contains:1- the synthetic Cloud which incorporates all the fuzzy opinions and reflects more general information coverage. 2- the weighted average Cloud which can be considered as the result of the group decision. 3- the decision maker's interval pair wise comparison which has been provided during the first phase and the decision maker's Cloud pair wise comparison which is mined from the context of the decision maker's interval pair wise comparison matrix. After having these feedback information, the decision makers can update their interval pair wise comparison matrices. These updated interval pair wise comparison matrices are presented in Table 5 and their corresponding Cloud matrices are presented in Table 6. From Table 5 we can see that the feedback information has helped the decision makers to update some of their opinions to be more reasonable. For example, Suleiman has updated his interval pair wise comparison between Homs and Swida'a from [7.5,7.9] to [7.2,7.6]. This new updated interval value is closer to the group's opinion and to the Cloud model extracted from the interval pair wise comparison matrix provided by Suleiman during the first phase.
4. The fourth step: is calculating the individual Cloud weight vectors from the Cloud matrices, then obtaining the final group Cloud weight vector by using the weighted geometric mean method. The Cloud weight vectors and the actual weight vector of our provinces are presented in Table 7.The actual areas of our provinces are: Homs 42223 Km2, Damascus 19631 Km2, Sweida'a 5550 Km2 and Dara'a 3730 Km2 and the actual relative sizes of these provinces are 0.594, 0.276, 0.0780 and 0.0524 respectively. According to Table 7, the final group Cloud weight vector is very close to the actual relative sizes. Moreover, all the actual relative sizes' membership degrees to their corresponding Cloud weights are not zero. That means, according to the group opinion, the actual relative sizes are potential relative sizes of our provinces.

Etab

	Homs	Damascus	Sweida'a	Dara'a
Homs	[1,1]	[1.8 , 2.1]	[7.3 , 7.6]	[10.7 , 11]
Damascus	[0.48,0.56]	[1,1]	[3 , 3.3]	[5.0 , 5.3]
Sweida'a	[0.13,0.14]	[0.30,0.33]	[1,1]	[1.4 , 1.5]
Dara'a	[0.09,0.09]	[0.19,0.20]	[0.67,0.71]]	[1,1]

ĈI= 0.0136, decision maker's importance weight=0.3332

Oula

	Homs	Damascus	Sweida'a	Dara'a
Homs	[1,1]	[1.6 , 2.4]	[6.6 , 8.3]	[10.8 , 11]
Damascus	[0.42,0.62]	[1,1]	[3.2 , 3.9]	[4.8 , 5.7]
Sweida'a	[0.12,0.15]	[0.26,0.31]	[1,1]	[1.2 , 1.8]
Dara'a	[0.09,0.09]	[0.18,0.21]	[0.56,0.83]	[1,1]

ĈI= 0.0189, decision maker's importance weight =0.3312

Suleiman

	Homs	Damascus	Sweida'a	Dara'a
Homs	[1,1]	[2.3 , 2.6]	[7.5 , 7.9]	[11 , 11]
Damascus	[0.43,0.45]	[1,1]	[3.6 , 3.8]	[5.2 , 5.5]
Sweida'a	[0.13,0.13]	[0.26,0.28]	[1,1]	[1.5 , 1.7]
Dara'a	[0.09,0.09]	[0.18,0.19]	[0.59,0.67]	[1,1]

ĈI= 0.0098, decision maker's importance weight=0.3347

Table 2. The interval pairwise comparison matrices of the first phase

Etab

	Homs	Damascus	Sweida'a	Dara'a
Homs	(1,0,0)	(2.0955,0.0459,0.0254)	(7.1401,0.1189,0.0969)	(10.6438,0.1629,0.079)
Damascus	(0.4825,0.0107,0.0054)	(1,0,0)	(3.4272,0.0749,0.0429)	(5.1195,0.0986,0.0733)
Sweida'a	(0.1418,0.0027,0.0023)	(0.2952,0.0061,0.0033)	(1,0,0)	(1.4996,0.0229,0.0127)
Dara'a	(0.0944,0.0015,0.0008)	(0.1972,0.0038,0.0029)	(0.6702,0.0098,0.0050)	(1,0,0)

Oula

	Homs	Damascus	Sweida'a	Dara'a
Homs	(1,0,0)	(2.0591,0.1208,0.0150)	(7.4308,0.4503,0.0855)	(10.9775,0.5592,0.2169)
Damascus	(0.5030,0.2970,0.0034)	(1,0,0)	(3.6943,0.2467,0.0578)	(5.4044,0.3065,0.0745)
Sweida'a	(0.1402,0.0087,0.0020)	(0.2850,0.0186,0.0039)	(1,0,0)	(1.4975,0.0882,0.0039)
Dara'a	(0.0954,0.0050,0.0019)	(0.1923,0.0109,0.0029)	(0.6900,0.0406,0.0019)	(1,0,0)

Suleiman

	Homs	Damascus	Sweida'a	Dara'a
Homs	(1,0,0)	(2.1604,0.0223,0.0107)	(7.6580,0.1034,0.0916)	(11.5962,0.1413,0.0879)
Damascus	(0.4642,0.0049,0.0024)	(1,0,0)	(3.5472,0.0557,0.0226)	(5.3803,0.0782,0.0671)
Sweida'a	(0.1314,0.0018,0.0016)	(0.2833,0.0047,0.0020)	(1,0,0)	(1.5194,0.0264,0.0096)
Dara'a	(0.0867,0.0009,0.0006)	(0.1872,0.0025,0.0022)	(0.6619,0.0112,0.0042)	(1,0,0)

Table 3. The Cloud pairwise comparison matrices of the first phase

synthetic Cloud matrix

	Homs	Damascus	Sweida'a	Dara'a
Homs		(2.105,0.1208,0.0170)	(7.4096,0.4503,0.0913)	(11.0725,0.5592,0.1280)
Damascus	(0.4832,0.0297,0.0037)		(3.5562,0.2467,0.0411)	(5.3014,0.3065,0.0716)
Sweida'a	(0.1378,0.0087,0.0020)	(0.2878,0.0186,0.0031)		(1.5055,0.08882,0.0088)
Dara'a	(0.0922,0.0050,0.0011)	(0.1922,0.0109,0.0027)	(0.6740,0.0406,0.0037)	

weighted average Cloud matrix

	Homs	Damascus	Sweida'a	Dara'a
Homs		(2.1029,0.0437,0.0104)	(7.4022,0.1589,0.0527)	(11.0614,0.1996,0.0823)
Damascus	(0.4828,0.0106,0.0023)		(3.5526,0.0878,0.0251)	(5.2961,0.1103,0.0414)
Sweida'a	(0.1377,0.0031,0.0012)	(0.2875,0.0067,0.0018)		(1.5040,0.0316,0.0055)
Dara'a	(0.0921,0.0018,0.0007)	(0.1920,0.0039,0.0015)	(0.6734,0.0144,0.0022)	

Table 4. Feedback information presented in tabular form

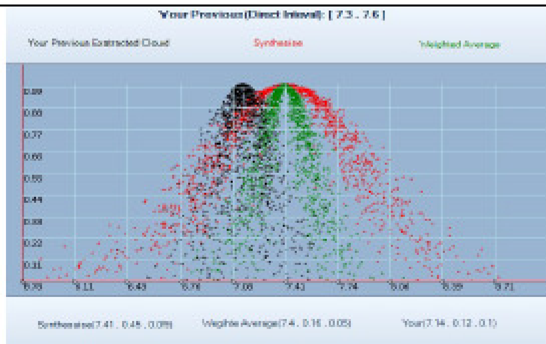
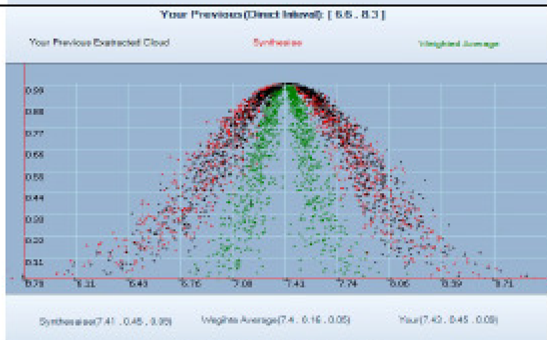
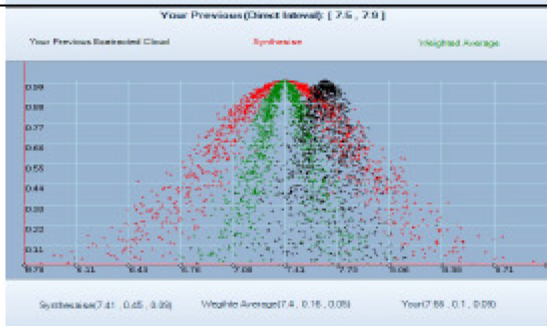
Etab	
Oula	
Suleiman	

Figure 4. Feedback information about the comparison between (2:Homs) and (3:Sweida’a)

Etab				
	Homs	Damascus	Sweida'a	Dara'a
Homs	[1,1]	[1.9 , 2.1]	[7.4 , 7.6]	[10.3 , 11]
Damascus	[0.48,0.53]	[1,1]	[3.1 , 3.4]	[5.0 , 5.2]
Sweida'a	[0.13,0.14]	[0.29,0.32]	[1,1]	[1.4 , 1.5]
Dara'a	[0.09,0.10]	[0.19,0.20]	[0.67,0.71]	[1,1]
CI= 0.0101, decision maker's importance weight =0.3337				
Oula				
	Homs	Damascus	Sweida'a	Dara'a
Homs	[1,1]	[1.7 , 2.5]	[6.6 , 8.3]	[10.8 , 11]
Damascus	[0.40,0.59]	[1,1]	[3.0 , 3.8]	[4.6 , 5.6]
Sweida'a	[0.12,0.15]	[0.26,0.33]	[1,1]	[1.2 , 1.8]
Dara'a	[0.09,0.09]	[0.18,0.22]	[0.56,0.83]	[1,1]
CI= 0.0190, decision maker's importance weight =0.3303				
Suleiman				
	Homs	Damascus	Sweida'a	Dara'a
Homs	[1,1]	[2.1 , 2.2]	[7.2 , 7.6]	[11 , 11]
Damascus	[0.45,0.48]	[1,1]	[3.5 , 3.7]	[5.0 , 5.4]
Sweida'a	[0.13,0.14]	[0.27,0.29]	[1,1]	[1.4 , 1.6]
Dara'a	[0.09,0.09]	[0.19,0.20]	[0.62,0.71]	[1,1]
CI= 0.0063, decision maker's importance weight=0.3351				

Table 5. The interval pairwise comparison matrices of the second phase

Etab				
	Homs	Damascus	Sweida'a	Dara'a
Homs	(1,0,0)	(2.1011,0.0373,0.0182)	(7.2192,0.1100,0.0726)	(10.5975,0.1608,0.0519)
Damascus	(0.4790,0.0084,0.0038)	(1,0,0)	(3.4464,0.0607,0.0298)	(5.0668,0.0852,0.0521)
Sweida'a	(0.1395,0.0023,0.0016)	(0.2921,0.0051,0.0023)	(1,0,0)	(1.4737,0.0224,0.0104)
Dara'a	(0.0946,0.0015,0.0005)	(0.1985,0.0034,0.0021)	(0.6811,0.0102,0.0045)	(1,0,0)
Oula				
	Homs	Damascus	Sweida'a	Dara'a
Homs	(1,0,0)	(2.1529,0.1289,0.0163)	(7.4458,0.4569,0.0921)	(11.0350,0.5667,0.2067)
Damascus	(0.4813,0.0288,0.0030)	(1,0,0)	(3.5431,0.2481,0.0466)	(5.2038,0.3079,0.0774)
Sweida'a	(0.1401,0.0088,0.0021)	(0.2978,0.0205,0.0035)	(1,0,0)	(1.5056,0.0926,0.0062)
Dara'a	(0.0949,0.0050,0.0017)	(0.2002,0.0119,0.0033)	(0.6885,0.0423,0.0024)	(1,0,0)
Suleiman				
	Homs	Damascus	Sweida'a	Dara'a
Homs	(1,0,0)	(2.1193,0.0245,0.0059)	(7.4778,0.1072,0.0346)	(11.0775,0.1442,0.0675)
Damascus	(0.4727,0.0055,0.0014)	(1,0,0)	(3.5342,0.0616,0.0201)	(5.2323,0.0858,0.0281)
Sweida'a	(0.1341,0.0019,0.0006)	(0.2824,0.0050,0.0017)	(1,0,0)	(1.4837,0.0280,0.0045)
Dara'a	(0.0906,0.0012,0.0006)	(0.1918,0.0031,0.0009)	(0.6765,0.0128,0.0021)	(1,0,0)

Table 6. The Cloud pairwise comparison matrices of the second phase

	Etab	Oula	Suleiman	Group	Actual
Homs	(0.584,0.010,0.005)	(0.587,0.035,0.008)	(0.589,0.008,0.003)	(0.587,0.022,0.006)	0.594
Damascus	(0.279,0.005,0.003)	(0.278,0.018,0.004)	(0.278,0.004,0.001)	(0.279,0.011,0.003)	0.276
Sweida'a	(0.081,0.001,0.001)	(0.081,0.005,0.001)	(0.079,0.001,0.001)	(0.080,0.003,0.001)	0.0780
Dara'a	(0.055,0.001,0.000)	(0.054,0.003,0.001)	(0.053,0.001,0.000)	(0.054,0.002,0.001)	0.0524

Table 7. Comparison between the final results and the actual relative area sizes.

4.2. Comparison between CDHA and Its Modified Version

In this section, to make the comparison, we firstly convert the interval pairwise comparison matrices presented in Table 5 into Cloud pairwise comparison matrices depending on the Cloud model building technique used by the original CDHA. Then, we calculate the consistency indices of these matrices by using the original CDHA consistency index calculation formula. After that, we count on these consistency indices in updating the decision maker's importance weights. Finally, we reckon the relative area sizes of our provinces (Table 9) depending on those matrices and importance weights (Table 8).

In Table 8, the cells that are shaded with dark gray contain fuzzily-useless Cloud pairwise comparisons. It is easy to notice that this table contains a lot of filled Clouds. Both of the filled and the fuzzily-useless Cloud pairwise comparisons lead to get fuzzily-useless and filled Clouds in the resulted weight vectors. These weight vectors are illustrated in Table 9. For example, the Cloud weight vector that is resulted from Suleiman's pairwise comparisons contains two fuzzily-useless Cloud weights which are the weights of Homs and Swida'a. The final group Cloud weight vector does not contain any fuzzily-useless Cloud, but its Cloud elements are thicker than the elements of the final group Cloud weight vector which is obtained by using the modified version of CDHA. For example the Cloud weight of Homs which is obtained by using the original CDHA is (0.587,0.016,0.009), while the Cloud weight of Homs which is obtained by using the modified version of CDHA is (0.587,0.022,0.006). These two Cloud weights have the same expectation, but the Cloud weight which is obtained by using the modified version of CDHA has a bigger En and a smaller He, which means that this cloud is thinner. As we mentioned before, from the point view of the fuzzy logic, such thin Cloud weights are more useful, because the membership degree of any possible weight number can be identified more accurately.

There is no big difference between the decision maker's importance weights which are calculated using the original CDHA (Table 8) and those which are calculated using the modified version of CDHA(Table 5), but the decision makers' ranks depending on these importance weights are different, i.e. if we order the decision makers by their weights which are calculated using the modified version of CDHA, we will get the following descending order, Suleiman > Etab> Oula. On the other hand, if we order them by their weights which are calculated using the modified version of CDHA, we will get the following descending order, Etab > Suleiman > Oula. The previous difference is due to the fact that the decision makers' importance weights are calculated depending on the consistency indices which are inaccurate in the original CDHA.

Etab				
	Homs	Damascus	Sweida'a	Dara'a
Homs	(1,0,0)	(2.0000,0.0333,0.0042)	(7.5000,0.0333,0.0583)	(10.6500,0.1167,0.0400)
Damascus	(0.5013,0.0084,0.0001)	(1,0,0)	(3.2500,0.0500,0.0098)	(5.1000,0.0333,0.0380)
Sweida'a	(0.1334,0.0006,0.0015)	(0.3083,0.0047,0.0003)	(1,0,0)	(1.4500,0.0167,0.0059)
Dara'a	(0.0940,0.0010,0.0004)	(0.1962,0.0013,0.0015)	(0.6905,0.0079,0.0020)	(1,0,0)
CI= 0.0046, decision maker's importance weight=0.3352				
Oula				
	Homs	Damascus	Sweida'a	Dara'a
Homs	(1,0,0)	(2.1000,0.1333,0.0187)	(7.4500,0.2833,0.1500)	(10.9000,0.0333,0.3789)
Damascus	(0.4941,0.0314,0.0049)	(1,0,0)	(3.4000,0.1333,0.0801)	(5.1000,0.1667,0.1244)
Sweida'a	(0.1360,0.0052,0.0033)	(0.2982,0.0117,0.0059)	(1,0,0)	(1.5000,0.1000,0.0130)
Dara'a	(0.0918,0.0003,0.0032)	(0.1980,0.0065,0.0052)	(0.6944,0.0463,0.0061)	(1,0,0)
CI= 0.0204, decision maker's importance weight=0.3293				
Suleiman				
	Homs	Damascus	Sweida'a	Dara'a
Homs	(1,0,0)	(2.1500,0.0167,0.0070)	(7.4000,0.0667,0.0323)	(11.0000,0.0000,0.1156)
Damascus	(0.4654,0.0036,0.0018)	(1,0,0)	(3.6000,0.0333,0.0296)	(5.2000,0.0667,0.0344)
Sweida'a	(0.1352,0.0012,0.0006)	(0.2780,0.0026,0.0025)	(1,0,0)	(1.5000,0.0333,0.0066)
Dara'a	(0.0909,0.0000,0.0009)	(0.1926,0.0025,0.0011)	(0.6696,0.0149,0.0029)	(1,0,0)
CI= 0.0063, decision maker's importance weight=0.3346				

Table 8. The original CDHA Cloud pairwise comparison matrices

	Original CDHA				The Modified version of CDHA
	Etab	Oula	Suleiman	Group	Group
Homs	(0.584,0.007,0.003)	(0.587,0.026,0.014)	(0.589,0.004,0.004)	(0.587,0.016,0.009)	(0.587,0.022,0.006)
Damascus	(0.279,0.004,0.001)	(0.278,0.013,0.006)	(0.278,0.003,0.002)	(0.279,0.008,0.004)	(0.279,0.011,0.003)
Sweida'a	(0.081,0.001,0.001)	(0.081,0.004,0.002)	(0.079,0.001,0.001)	(0.080,0.002,0.001)	(0.080,0.003,0.001)
Dara'a	**(0.055,0.001,0.000)**	(0.054,0.002,0.001)	(0.053,0.001,0.000)	(0.054,0.002,0.001)	(0.054,0.002,0.001)

Table 9. Comparison between the results of the original and the modified version of CDHA

5. Conclusion And Future Work

In spite of its strength, CDHA has two defects. These defects lie in the techniques used in defining the consistency index and building the Cloud pairwise comparisons. To avoid these defects, a new modified version of CDHA has been proposed and discussed. This version assigns valid importance weights to the decision makers and leads to get more rational Cloud weights, which give more rational group opinion. In addition to that, the resulted Cloud weights of the modified version of CDHA are thinner than those we get by using the original one. Such thin Cloud weights, from the point view of the fuzzy logic, are more useful, because the membership degree of any possible weight number can be identified more accurately. A simple case study that involved reproducing the relative area sizes of four provinces in Syria has been used to illustrate the modified version and to compare it with the original one.

Our future work will be upgrading the aggregation technique used by the modified version of CDHA, which is the traditional AHP linear technique. This technique calculates the alternative final weight by aggregating its weights on each criterion in a linear manner. The upgraded technique could be implemented by converting the resulted alternative's thin Cloud weights on each criterion into Type2 fuzzy membership functions and aggregating these membership functions by using a Type2 Fuzzy Rule Base System.

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