A BLIND ROBUST WATERMARKING SCHEME BASED ON SVD AND CIRCULANT MATRICES

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ABSTRACT

Multimedia security has been the aim point of considerable research activity because of its wide application area. The major technology to achieve copyright protection, content authentication, access control and multimedia security is watermarking which is the process of embedding data into a multimedia element such as image or audio, this embedded data can later be extracted from, or detected in the embedded element for different purposes. In this work, a blind watermarking algorithm based on SVD and circulant matrices has been presented. Every circulant matrix is associated with a matrix for which the SVD decomposition coincides with the spectral decomposition. This leads to improve the Chandra algorithm [1], our presentation will include a discussion on the data hiding capacity, watermark transparency and robustness against a wide range of common image processing attacks.

KEYWORDS

Digital image watermarking, Singular value Decomposition, circulant matrix, ownership protection.

1. INTRODUCTION

Due to improvements in the digital image technology and growing availability and usability of internet during the past several years, demands for storage and transmitting of digital images have seen a distinct increase, Unfortunately, the problem of illegal piracy is increasingly serious. Protection of digital multimedia content has become an increasingly important issue for content owners and service providers. Encryption data was a way to ensure only the owner to view the content, there are still ways for illegal using of the content after decryption [2, 3], that lead us to a new method for protection. Watermarking is the process of embedding data called a watermark into the multimedia object such that watermark can be detected or extracted later to make an assertion about the object. The object may be an audio, image or video and even 3-D models [4, 5]. Watermarking algorithms fall into two categories. The first form of watermarking was a spatial watermarking technique work with the pixel values directly. Generally, spatial domain watermarking is easy to implement from a computational point of view, but too fragile to resist numerous attacks [6, 7]. In spatial domain, the watermark is directly embedded into the specific pixels of the host image, but in transform domain which our proposed scheme based in the watermark is embedded into the transform coefficients of the host image after applying DWT, DFT, DCT or SVD transform, and this called the frequency domain watermarking. Because of the

weakness of the spatial domain watermarks, watermarking in the frequency domain became more attractive as a result of its higher robustness against attacks compared to the spatial domain watermarking. To this aim a number of robust methods based on the SVD transform were introduced but these methods didn't offer good transparency and robustness against geometric attacks. Starting with Liu and Tan [8] an image watermarking method based on SVD, this method is robust against some attacks, and it is a non-blind method and it has a weak imperceptibility, Chandra et al. [1] also introduced a digital image watermarking method. This method is based on moderate modifying of the singular values of the host image. This method is weak against geometric attacks. Ganic et al. [9] proposed a method based on SVD in discrete wavelet transform (DWT) domain. The insertion procedure concerns the modifying of the singular values of the wavelet transformed sub-bands with the singular values of the mark. This scheme is a non-blind and the transparency of the watermarked image is weak. Makhloghi et al.[10] also proposed a scheme based on singular value decomposition in wavelet domain for copyright protection but his method lacks to robustness. Lin et al. [11] presented a full-band DWT domain image watermarking method based on SVD. This method has good robustness against common attacks but its drawback is that the original image is required in extraction procedure. Also the quality of the watermarked image is not good. The watermarking algorithms described in [1-8] are semi or non blind. Soumya Mukherjee and Arup Kumar Pal [12] proposed a robust watermarking scheme which employs both the Discrete Cosine Transform (DCT) and the Singular Value Decomposition (SVD), It starts with transformation of an original image into a transformed image using block based DCT. From each transformed block, the middle band DCT coefficients are selected to form a reduced transformed image and then the watermark is embedded into the constructed reduced transformed image after a suitable SVD operation. This method has good robustness against different attacks but it has a weak imperceptibility. In this paper, a new SVD-based method is proposed which gives the variety in creating the watermark under (1, 2, ..., n) blocks depending on the initiate coefficients and using the circulant's matrix properties to make Chandra algorithm [1] blind instead of non blind and turn it robust against different geometric and non geometric attacks. Organization of the paper is as follows: Section 2 explains the concept of SVD and Circulant matrices while Section 3 presents the proposed method. Section 4 throws light on the experimental results and a comparative analysis of our scheme and other schemes is given, whereas the summary of results and the conclusion is presented in Section 5.

2. SINGULAR VALUE DECOMPOSITION AND CIRCULANT MATRICES

2.1. Singular value decomposition SVD

Every real matrix A can be decomposed into a product of three matrices :

$$A = U \times S \times V^t \tag{1}$$

where U and V are orthogonal matrices such that $U \times U^t = I$ and $V \times V^t = I$ where I is the Identity matrix and S is the diagonal matrix, $S = diag(\partial_1, \partial_2 \cdots)$ with $\partial_1 \geq \partial_2 \geq \cdots \geq 0$ The diagonal entries ∂_i of S are called the singular values of A, they are the eigenvalues of $A \times A^t$ or $A^t \times A$. The columns of U are the left singular vectors of A, they are eigenvectors of $A \times A^t$ and the columns of V called the right singular vectors of A and they are eigenvectors of $A^t \times A$.

2.2. Circulant matrices

The circulant matrix $C = cir(c)$ associated to the vector $c \in R^n$ is the $n \times n$ matrix whose rows are given by iterations of the shift operator T acting on c, its Kth row is $T^k c$, $k = 1,..,n$

For example if $c = (c_1, c_2, c_3, c_4)$, the 4×4 circulant matrix

$$C = cir(c) \text{ is giving by } \begin{pmatrix} c_1 & c_2 & c_3 & c_4 \\ c_4 & c_1 & c_2 & c_3 \\ c_3 & c_4 & c_1 & c_2 \\ c_2 & c_3 & c_4 & c_1 \end{pmatrix} \tag{2}$$

3. PROPOSED METHOD

We consider a circulant matrix $c = cir(c_1, c_2, c_3, c_4)$

The matrix $CC^t = C^t C$ is positive symmetric matrix so its spectral decomposition coincides with its SVD decomposition that is $CC^t = U_0 diag(\delta_1, \delta_2, \delta_3, \delta_4) U_0^t$ with

$$\delta_1 = (c_1 + c_2 + c_3 + c_4)^2$$
$$\delta_2 = (c_1 - c_2 + c_3 - c_4)^2 \tag{3}$$
$$\delta_3 = \delta_4 = (c_1 - c_3)^2 + (c_2 - c_4)^2$$

are the singular values and U_0 is the constant matrix :

$$U_0 = \begin{pmatrix} 1/2 & -1/2 & 0 & -\sqrt{2}/2 \\ 1/2 & 1/2 & -\sqrt{2}/2 & 0 \\ 1/2 & -1/2 & 0 & \sqrt{2}/2 \\ 1/2 & 1/2 & \sqrt{2}/2 & 0 \end{pmatrix} \tag{4}$$

If A is an image of size $4m \times 4m$, to every vector $c = (c_1, c_2, c_3, c_4)$ is associated a 4×4 circulant matrix $C_1 = cir(c_1)$ and a watermark as $4m \times 4m$ matrix with one block

$$W_1 = \begin{pmatrix} C_1 C_1^t & 0 & . & 0 \\ 0 & 0 & & . \\ . & & . & . \\ 0 & & . & 0 \end{pmatrix} \tag{5}$$

To obtain a watermark W_k with k blocks

$$W_k = \begin{pmatrix} C_1C_1^t & 0 & . & . & & . & 0 \\ 0 & C_2C_2^t & & & & & . \\ . & & . & & & & . \\ . & & & C_kC_k^t & & & . \\ & & & & 0 & & \\ . & & & & & . & . \\ 0 & . & . & . & & . & 0 \end{pmatrix} \tag{6}$$

3.1. Watermark insertion procedure

To watermark a given original image A of size $4m \times 4m$ we will use a watermark with one block as following:

1. Take $c = (c_1, c_2, c_3, c_4)$ such that $\partial_4 \geq \partial_3 \geq \partial_2 \geq \partial_1$
2. Construct the watermark of size $4m \times 4m$

$$W_1 = \begin{pmatrix} C_1C_1^t & 0 & . & 0 \\ 0 & 0 & & . \\ . & & . & . \\ 0 & & . & 0 \end{pmatrix} \tag{7}$$

3. Apply SVD on A :
$$A = U \times S \times V^t \text{ with } S = diag(S_i) \tag{8}$$

4. Apply SVD on W_1 :

$$W_1 = \begin{pmatrix} U_0 & 0 \\ 0 & I \end{pmatrix} \begin{pmatrix} \partial & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} U_0^t & 0 \\ 0 & I \end{pmatrix} \tag{9}$$

With $\partial = diag(\delta_1^1, \delta_2^1, \delta_3^1, \delta_4^1)$ and I is $4(m-1) \times 4(m-1)$ identity matrix.
   Put
$$Y_i = S_i + \alpha \partial_i' \tag{10}$$

with $\partial_1' = \delta_1^1$, $\partial_2' = \delta_2^1, \partial_3' = \delta_3^1$, $\partial_4' = \delta_4^1$ and $\forall i > 4 \ \partial_i' = 0$
So
$$A^* = U \times diag(Y_i) \times V^t \tag{11}$$

is the watermarked image.

3.2. Watermarking detection and extraction procedure

We don't require the original image A to detect the watermark, we only require the watermarked image A^*, the scaling factor α and the key $S_i = (S_1, S_2, S_3, S_4)$ formed by the first four values of S .

1. Apply SVD to A^*

$$A^* = U^* \times S^* \times V^{*t} \tag{12}$$

2. Calculate

$$x_i = \frac{S_i^* - S_i}{\alpha} \tag{13}$$

for the first four elements.

If $x_3 = x_4$ then the mark is detected else the watermark is not present on the image.
To extract the mark we compute:

$$W^* = \begin{pmatrix} U_0 & 0 \\ 0 & I \end{pmatrix} \begin{pmatrix} X & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} U_0^t & 0 \\ 0 & I \end{pmatrix} \tag{14}$$

where $X = diag(x_1, x_2, x_3, x_4)$ and I is $4(m-1) \times 4(m-1)$ identity matrix.

Remarks:

1. If we use a watermark W_k with k blocks, to detect or extract the watermark we only require the scaling factor α and a key $K_2 = (S_1, ..., S_{4k})$ of length 4k which contains the first 4k values of S. In this case the sequence $X = (x_i)$ is of length 4k and the mark is detected if $x_{4i-1} = x_{4i}$ for $i = 1, ..., k$.

2. In Chandra algorithm [1], to extract the watermark (W), U_w, V_w are required, in the proposed scheme $U_w = V_w$ is a constant matrix and independent of the watermark, thus our proposed algorithm is blind.
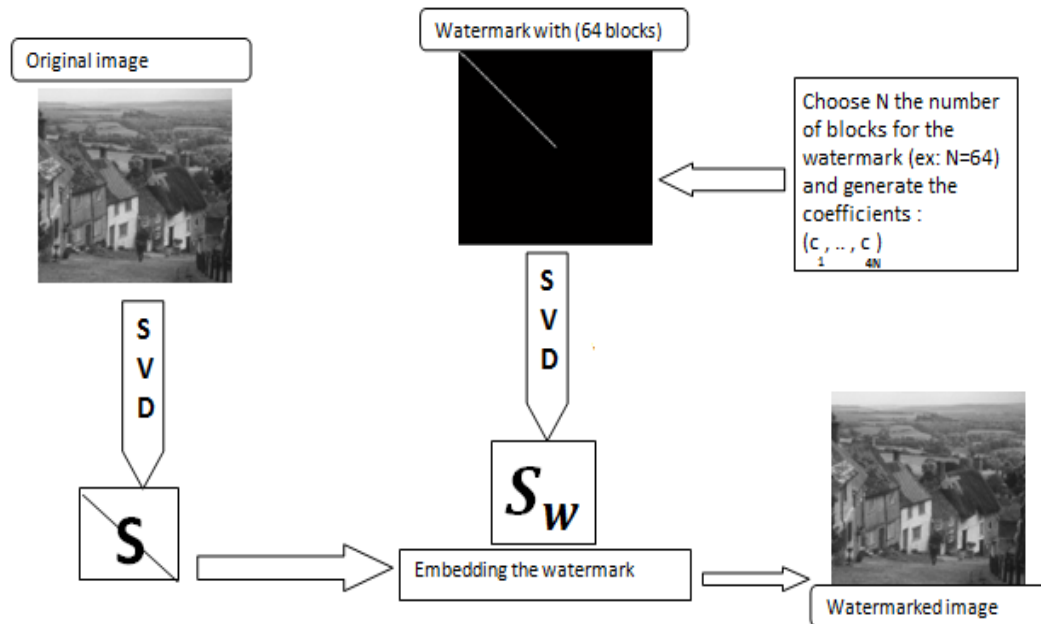

Figure 1. The Proposed watermarking scheme.

Our proposed watermarking method can be concluded in (Figure 1). As it is shown to construct the watermark we first choose N the number of blocks of the watermark then we generate the

coefficients ($c_1,..,c_{4N}$) and by following the steps mentioned above we embed the watermark into the original image.

4. EXPERIMENTAL RESULTS

The proposed scheme is implemented using MATLAB.   Six 512×512 images Lena , Goldhill, Baboon, Barbara, Peppers, Boat were used in the simulation (figure 2). The signature is a 512 × 512 image composed with N blocks in its diagonal.

Figure 2. Original test images

To evaluate the quality of the watermarked images we use the PSNR measure defined as:

$$PSNR = 10\log_{10}(\frac{255^2}{MSE})db \tag{15}$$

With *MSE* is the mean squared error between the original and watermarked image. The PSNR values of the watermarked images by our method indicate that our method in general achieves very good quality as it shown in (Table 1).

Table 1.   Relationship of number of blocks of the watermark and PSNR of different watermarked images. (dB) Sf sets to 0.06

Nb blocks / Image name	1	3	5	10	30	64	80	100	128
Lena	56.6994	55.5982	55.7382	55.0831	55.8089	56.0630	55.4090	55.4366	55.3805
Goldhill	57.2825	55.5499	55.4955	55.1460	54.6665	54.6762	52.9705	52.8710	52.8454
Baboon	55.6458	54.6902	55.0781	55.1058	51.6568	50.4369	49.4199	49.1576	49.2020
Barbara	56.9139	55.5102	57.1737	55.2064	53.3403	51.5035	50.2628	50.1065	50.0764
Peppers	55.9094	55.2634	55.1153	53.5932	54.5946	55.4747	56.2183	56.2190	56.2759
Boat	57.1737	55.6458	55.2064	54.6902	51.5035	52.9705	52.8710	52.8454	52.8710

Its clear from (Table1) that the proposed method preserves good transparency for the watermarked images.
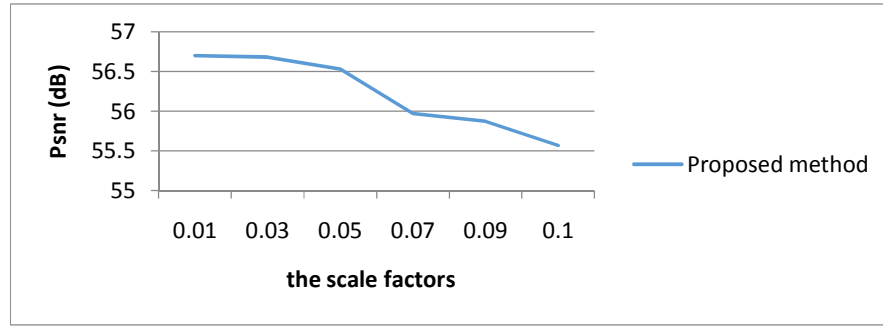
Figure 3. Relationship of the scale factors (α) and PSNR for the proposed method

(Figure 3) shows a relation between α and transparency in terms of the PSNR value. We can notice that our method has a good PNSR values and it reach its optimal value when the scale factors (α) is between 0.01 and 0.03.

Figure 4. Watermarked images

(Figure 4) shows the watermarked images used in the simulation, it can be seen that there is no perceptual difference between original and watermarked images that is also supported by good PSNR value. It shows that the good imperceptibility is obtained by proposed technique.

We use normalized correlation (NC) to evaluate the quality of the extracted watermark, and this measure is defined by:

$$NC(W,W') = \frac{1}{W_h \times W_w} \sum_{i=0}^{W_h-1} \sum_{j=0}^{Ww-1} W(i,j) \times W'(i,j) \qquad (16)$$

with W_h and W_w are the height and width of the watermarked image ,respectively. $W(i,j)$ and $W'(i,j)$ denote the coefficients of the inserted watermark and the extracted watermark respectively.

As we can see in (Table 2) our proposed method is robust against different geometric and non-geometric attacks, and we can notice also that the relationship between the number of blocks of the watermark and the resulted NC generally are inversely proportional except for the rotation attack.

Table 2. Resulted NC for different image attacks using variation of watermarks.

Attacks	Jpeg			speckle	imsharpen	Smooth	Rotation		salt & pepper	FFT	Filtre median	translate	Gaussian filter
Nb blocks	50	60	90	0.04			3°	5°	0.02			[20 35]	hsize = [5 5] sigma = 2
1	0.9941	0.9941	0.9941	0.9985	0.9996	0.9993	0.9684	0.8477	0.9998	0.9941	1.0000	0.9989	0.9994
3	0.9577	0.9577	0.9578	0.9952	0.9982	0.9982	0.9517	0.8560	0.9997	0.9581	0.9998	0.9941	0.9974
5	0.9236	0.9236	0.9238	0.9928	0.9972	0.9975	0.9471	0.8634	0.9996	0.9246	0.9993	0.9906	0.9947
10	0.8596	0.8598	0.8603	0.9900	0.9931	0.9941	0.9454	0.8822	0.9993	0.8623	0.9962	0.9882	0.9756
30	0.7551	0.7559	0.7584	0.9885	0.9830	0.9489	0.9582	0.9195	0.9985	0.7656	0.9615	0.9889	0.8280
64	0.7235	0.7249	0.7280	0.9867	0.9796	0.8864	0.9711	0.9423	0.9978	0.7373	0.9226	0.9911	0.7058
80	0.7102	0.7123	0.7143	0.9825	0.9782	0.8459	0.9740	0.9465	0.9961	0.7243	0.8979	0.9927	0.6360
100	0.7082	0.7107	0.7121	0.9807	0.9781	0.8389	0.9755	0.9487	0.9952	0.7220	0.8939	0.9929	0.6243
128	0.7072	0.7100	0.7115	0.9803	0.9781	0.8371	0.9757	0.9491	0.9945	0.7213	0.8924	0.9928	0.6214

Table 3. Comparison of PSNR for Lai et al. [13], Tsai et al [14] and our algorithm.

Method	the scale factors				
	0.01	0.03	0.05	0.07	0.09
Lai et al .[13]	51.14	51.14	50.89	49.52	47.49
Tsai et al [14]	47	37	33	28	about 25
Proposed method	56.70	56.68	56.53	55.97	55.87

(Table 3) shows the comparison of PSNR for two other algorithms and our algorithm. The values of the scale factors, (SFs) are carried out with constant range from 0.01 to 0.09 with an interval of 0.02. Size of host images are 256×256 for Lai et al. [13], and 512×512 for Tsai et al. [14] and our scheme.

Table 4.   The comparison of robustness and imperceptibility (dB) for our scheme and Soumya et al. [12] under various image processing attacks.

Attack	NC of the extracted watermark (Proposed Scheme)	PSNR of the Attacked watermarked image (Proposed Scheme)	NC of the extracted watermark [12]	PSNR of the Attacked watermarked image [12]
Gaussian Lowpass Filtering (3×3)	**0.9991**	**56.1734 dB**	0.9974	41.2799dB
Average Filtering	**0.9946**	**52.6980 dB**	0.8253	35.7368 dB
Image Noising by salt-and-pepper noise	**0.9927**	**50.1413 dB**	0.9009	24.7876 dB

image enhancement by histogram equalization	0.9122	**40.0745 dB**	**0.9254**	12.1774 dB
Center-cropped attack (*64 ×64* pixels) and filled with pixel value *0*	0.9178	**41.4920 dB**	**0.9417**	20.8188 dB
Center-cropped attack (*64 ×64* pixels) and filled with pixel value *255*	**0.9582**	24.7561 dB	0.8983	15.0842 dB
Center-cropped attack (*128 ×128* pixels) and filled with pixel value 0	0.8793	**26.3998 dB**	**0.8979**	15.3449 dB
Center-cropped attack (*128 ×128* pixels) and filled with pixel value *255*	**0.9577**	**13.8987 dB**	0.8743	8.8648 dB
JPEG Compression (QF=25)	**0.9979**	**55.4146 dB**	0.9281	35.9800 dB
JPEG Compression (QF=50)	0. 9979	**55.4132 dB**	**1**	38.7407 dB

The bold values indicates the best values comparing with the others.

(Table 4) presents the comparison of robustness and imperceptibility (dB) for our scheme and Soumya et al. [12] under various image processing attacks, besides the robustness results the proposed scheme achieved high imperceptibility compared with Soumya et al. [12

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(a) Result under Gaussian Lowpass filtering (3×3) attack

(b) Result under Average filtering attack

(c) Result under salt-and-pepper noise attack

(d) Result under histogram equalization attack

(e) Result under Center-cropped attack (*64×64* pixels) and
       filled with
       pixel value *0*

(f) Result under Center-cropped attack (*64×64* pixels) and filled with pixel value *255*

(g) Result under Center-cropped attack (128×128 pixels) and filled with pixel value 0

(h) Result under Center-cropped attack (*128×128* pixels) and filled with pixel value *255*

(i) Result under JPEG Compression (QF=50) attack
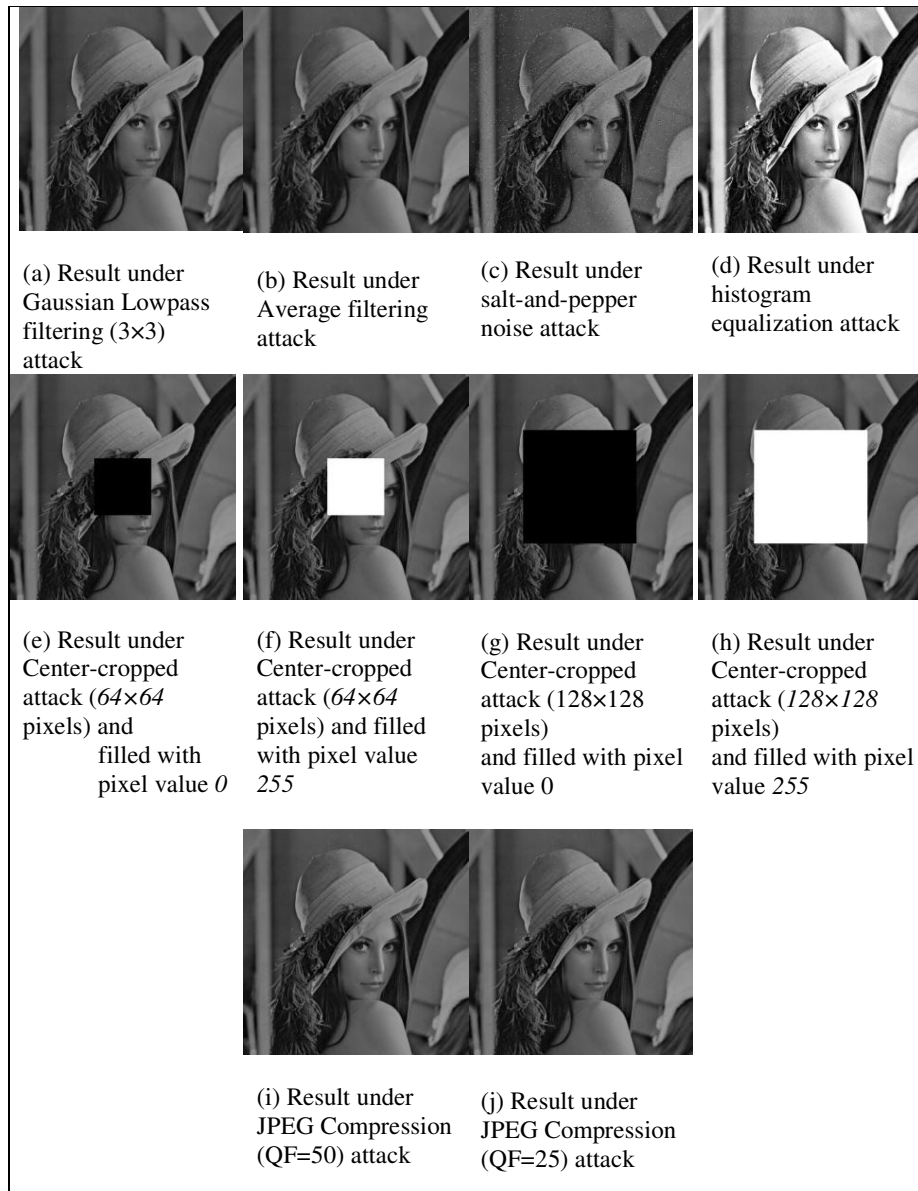
(j) Result under JPEG Compression (QF=25) attack

Figure 5. The attacked watermarked image under various image processing attacks in the comparison with Soumya et al. [12]

(Figure 5) shows the attacked images of the comparison with the scheme of Soumya et al.[12].
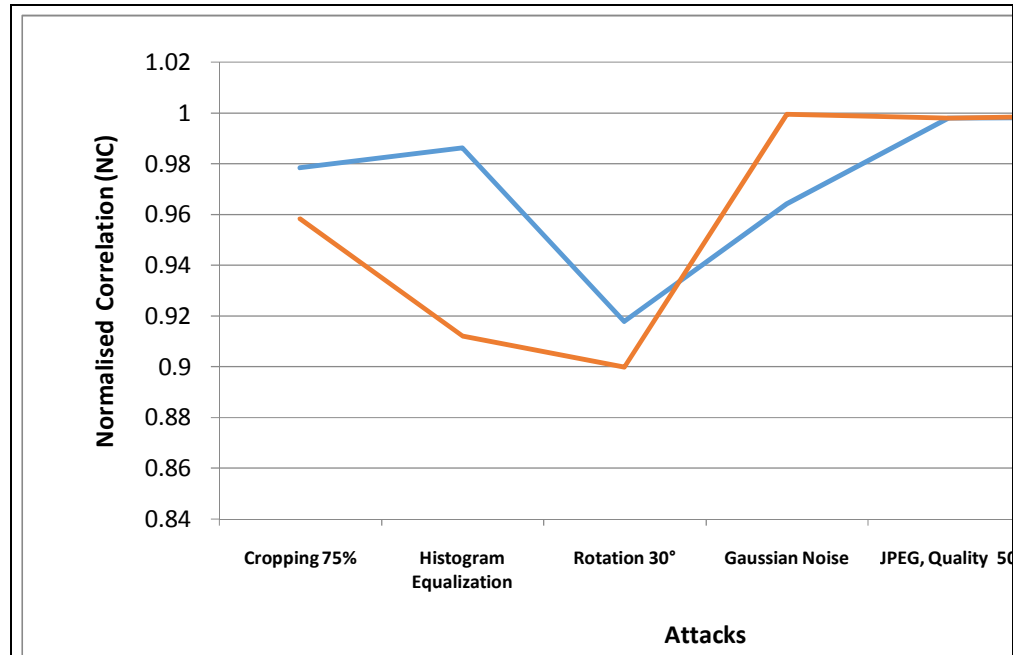
Figure 6.   Comparison of our algorithm and Musrat et al [15] in term of *NC* values

(Figure 6) presents a comparison of robustness between the proposed method and Musrat et al [15] method, it shows that Musrat's scheme is more robust against such attacks as cropping 75% , histogram equalization and rotation 30° compared to our method but ours still achieves a good NC values after applying those attacks, and the minimum value of NC was 0.9 which means its robust, and for the other attacks (Gaussian noise, JPEG compression and Translation) our method perform better than Musrat's scheme.

Table 5**.** The comparison of robustness for our scheme, Nasrin et al [17] , Lai et al. [13] and Rastegar et al. [16].

Attack	Proposed scheme	Nasrin et al [17]	Lai et al .[13]	Rastegar et al.[16] [a]	Rastegar et al.[16] [b]
Pepper & salt noise (0.3)	**0.9927**	0.8926	–	0.7515	0.8258
Speckle noise (var=0.01)	**0.9950**	0.952	–	0.9609	0.9667
Gaussian noise (M=0,var=0.5)	**0.9210**	0.8935	–	0.7926	0.82
Gaussian filtering (3 ×3)	**0.9990**	0.987	–	0.8023	0.9843
Median filtering (3×3)	**0.9885**	0.982	0.9597	0.7534	0.9706
Wiener filtering (3×3)	0.9826	**0.984**	–	0.9824	0.9569
Sharpening	**0.9966**	0.932	–	0.9687	0.9511
Histogram equalization	0.9122	**0.990**	0.9862	0.9648	0.9628
Gamma correction (0.7)	0.9887	0.9935	**0.9982**	–	
Gamma correction (0.8)	0.9890	**0.9950**	–	0.7203	0.9217
JPEG compression Q = 30	**0.9937**	0.987	–	–	–
JPEG compression Q = 10	**0.9915**	0.972	0.9772	0.9824	0.9843
JPEG compression Q = 5	**0.9907**	0.952	–	0.8532	0.9354
Scaling zoom(out = 0.5, in = 2)	**0.9772**	0.948	–	0.5127	0.953
Rotation (angle = 2°)	0.9648	**0.981**	–	0.5068	0.9628
Rotation (angle =−30°)	0.8532	**0.9823**	0.9780	–	–

'**–**' means the attacks are not done.
The bold values indicates the best values comparing with the others.
a Indicates the first scheme in Rastegar et al. [16]
b Indicates the second scheme in Rastegar et al.[16]

(Table 5) shows the proposed results with Rastegar's schemes results and Lai's scheme results and Nasrin et al's scheme when scaling factor is 0.05. Rastegar scheme (a) represents the embedding in all sub-bands while Rastegar scheme (b) represents the embedding in LH and HL only. Our scheme performed better than Nasrin's , Lai's and Rastegar's schemes as shown in (Table 5) .

5. CONCLUSIONS

This paper presents a blind robust digital image watermarking scheme based on singular value decomposition and on circulant matrix for copyright protection, using the circulant matrix's properties we improved the algorithm of Chandra and turned it to a blind watermarking algorithm after it was a non blind in addition to the augmentation of its robustness. Simulation results indicate that the proposed method achieves higher robustness compared to other known watermarking methods. The proposed method is robust against a wide range of common image processing attacks.

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