ON OBSERVER DESIGN METHODS FOR A CLASS OF TAKAGI-SUGENO FUZZY SYSTEMS

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ABSTRACT

The generalized design principle of TS fuzzy observers for one class of continuous-time nonlinear MIMO systems is presented in this paper. The problem addressed can be indicated as a descriptor system approach to TS fuzzy observers design, implying the asymptotic convergence of the state observer error. A new structure of linear matrix inequalities is outlined to possess the observer asymptotic dynamic properties closest to the optimal.

KEYWORDS

Thau observer, TS fuzzy observer, convex optimization, linear matrix inequalities

1. INTRODUCTION

As is well known, observer design is a hot research field owing to its particular importance in observer-based control, and fault diagnosis. The nonlinear system theory using Lipchitz conditions has emerged as a method capable of use in state estimation design for nonlinear systems [1], although Lipschitz condition is a restrictive condition which many classes of systems may not satisfy. However this principle used in state estimators design means that the observer satisfies a sufficient condition for the asymptotic stability of error system, but in fact there is not a straightforward method for selecting the observer gain to fill such conditions [2]. Because the Takagi-Sugeno (TS) fuzzy approach is a suitable representation of certain class of nonlinear dynamic systems [3], employing the fuzzy modelling approach to approximate sector-bounded nonlinear systems, other well-known nonlinear observers are based on Takagi-Sugeno (TS) fuzzy models [4], [5]. To design TS fuzzy observers, usually the technique utilizing the linear matrix inequalities is used [6].

Although the state observers for linear and nonlinear systems received considerable attention, the descriptor design principles have not been studied extensively. Adapting the descriptor observer design principle [7], the first result giving sufficient design conditions, but for linear time-delay systems, can be found in [8]. Reflecting the same problems concerning the observers for descriptor time-delay nonlinear systems represented by TS fuzzy models, an LMI method was presented in [9], but a hint of this methodology can be found only in [10].

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Adapting the results on the TS fuzzy observers for bilinear systems [11] as well as their potential extensions, the main issue of this paper is to use the descriptor principle in TS fuzzy observer design. Preferring LMI formulation, although partly conservative, the stability condition proofs use standard arguments on H_2 approach to obtain the design conditions requiring only solving of LMIs without additional constraints. To the best author's knowledge, the proposed LMI structure in design conditions formulation were not fully addressed yet in the previous works.

The paper is organized as follows. In Sec. 2, the TS fuzzy model is briefly described and the TS fuzzy observer design problem for given class of nonlinear systems is formulated in Sec. 3. The new LMI structure, describing the TS fuzzy observer design conditions, is presented in Sec. 4 and analysed and algorithmically explained in Sec 5. Finally, Sec. 6 draws conclusions and some future directions.

The notations throughout the paper are narrowly standard in such a way that $\boldsymbol{x}^T$, $\boldsymbol{X}^T$ denotes the transpose of the vector $\boldsymbol{x}$ and matrix $\boldsymbol{X}$, respectively, $\boldsymbol{X} = \boldsymbol{X}^T > 0$ means that $\boldsymbol{X}$ is a symmetric positive definite matrix, the symbol $\boldsymbol{I}_n$ indicates the n-th order unit matrix, R denotes the set of real numbers R^n, and $R^{n\times r}$, refer to the set of all n-dimensional real vectors and $n\times r$ real matrices, respectively.

2. TAKAGI-SUGENO FUZZY MODELS

The systems under consideration are from the class of multi-input and multi-output nonlinear (MIMO) continuous-time dynamic systems, represented in TS form as

$$\dot{\boldsymbol{q}}(t) = \sum_{i=1}^{s} \mathrm{h}_i(\boldsymbol{\theta}(t))(\boldsymbol{A}_i\boldsymbol{q}(t) + \boldsymbol{B}_i\boldsymbol{u}(t)) \tag{1}$$

$$y(t) = Cq(t) \tag{2}$$

where $q(t)\in R^n$, $u(t)\in R^r$, $y(t)\in R^m$, are vectors of the state, input, and output variables, $A\in R^{n\times n}$, , $C\in R^{m\times n}$ are real finite values matrices, $m, r < n$ and $\mathrm{h}_i(\theta(t))$ is averaging weight for the i-th rule, representing the normalized grade of fuzzy membership (membership function). By definition, the membership functions satisfy the convex sum properties

$$0 \le \mathrm{h}_i(\theta(t)) \le 1, \quad \sum_{i=1}^{s}\mathrm{h}_i(\theta(t)) = 1 \quad \textit{for all } i \in \langle 1,\ldots,s\rangle \tag{3}$$

where s is the number of linear models (fuzzy rules) and

$$\theta(t) = \begin{bmatrix}\theta_1(t) & \theta_2(t) & \mathrm{L} & \theta_p(t)\end{bmatrix} \tag{4}$$

is p dimensional vector of the premise variables. It is assumed that the premise variable is a system state variable, or a measurable external variable, while none of the premise variables does not depend on any element of the input variables vector $u(t)$. In the above sense, the fuzzy model of a system can be interpreted as a combination of s linear models through the set of membership functions $\{h_i(\theta(t)), i = 1,2,\ldots,s\}$. More details can be found, e.g., in [6], [12].

It is supposed that the couples (A_i, C) are observable for all $i = 1,2,\ldots,s$, as well as the matrix C occurs in all local models and the number of input variables r is equal to the number of output variables m (the dynamic system is a square system).

3. TAKAGI-SUGENO FUZZY OBSERVER DESIGN

The conventional fuzzy observer can be constructed as follows

$$\dot{\boldsymbol{q}}_e(t) = \sum_{i=1}^{s} h_i(\boldsymbol{\theta}(t))(\boldsymbol{A}_i\boldsymbol{q}_e(t) + \boldsymbol{B}_i\boldsymbol{u}(t) + \boldsymbol{J}_i(\boldsymbol{y}(t) - \boldsymbol{y}_e(t)) \tag{5}$$

$$y_e(t) = Cq_e(t) \tag{6}$$

where $q_e(t) \in R^n$ is estimation of the system state vector (the fuzzy observer state vector) and $J_i \in R^{n\times m}, i = 1,2,\ldots, s$ is the set of the observer gain matrices.

Lemma 1

The fuzzy observer (5), (6) is stable if there exist a positive definite symmetric matrix $P \in R^{n\times n}$ and matrices $Y_i \in R^{n\times m}$ such that for all $i = 1,2,\ldots,s$

$$P = P^T > 0 \tag{7}$$

$$PA_i + A_i^T P - Y_i C - C^T Y_i^T < 0 \tag{8}$$

When the above conditions hold, i.e. if Y_i and the non-singular matrix P are solutions of (7), (8), the set of the observer gain matrices J_i is given by the following equations

$$J_i = P^{-1}Y_i \tag{9}$$

Proof: (compare, e.g., [11]) Introducing the error between the system state vector and the observer state vector as follows

$$e(t) = q(t) - q_e(t) \tag{10}$$

and performing the time derivative of the error $e(t)$, then exploiting (1) and (10) it is

$$\dot{\boldsymbol{e}}(t) = \dot{\boldsymbol{q}}(t) - \dot{\boldsymbol{q}}_e(t) = \sum_{i=1}^{s} h_i(\boldsymbol{\theta}(t))(\boldsymbol{A}_i(\boldsymbol{q}(t) - \boldsymbol{q}_e(t)) - \boldsymbol{J}_i(\boldsymbol{y}(t) - \boldsymbol{y}_e(t))) \tag{11}$$

which can be written using (2), (11) as follows

$$\dot{\boldsymbol{e}}(t) = \sum_{i=1}^{s} h_i(\boldsymbol{\theta}(t))\boldsymbol{A}_{ei}\boldsymbol{e}(t) \tag{12}$$

where

$$A_{ei} = A_i - J_iC \tag{13}$$

Defining the Lyapunov function of the form

$$v(e(t)) = e^T(t)Pe(t) > 0 \tag{14}$$

where $P = P^T > 0$, then evaluating the time derivative of (14) it yields

$$\dot{v}(e(t)) = \dot{e}^T(t)Pe(t) + e^T(t)P\dot{e}(t) \tag{15}$$

Substituting (12), (13) into (15) gives

$$\dot{v}(e(t)) = e^T(t)\sum_{i=1}^{s} h_i(\theta(t))(P(A_i - J_iC) + (A_i - J_iC)^T P)e(t) \tag{16}$$

$$P(A_i - J_iC) + (A_i - J_iC)^T P < 0 \text{ for all } i \tag{17}$$

respectively.
Therefore, setting

$$PJ_i = Y_i \tag{18}$$

(17) implies (8). This concludes the proof.

Considering the affine properties of the TS fuzzy models, to reduce the conservatism in solution the enhanced design criterion can be derived by using two slack matrices.

Theorem 1

The fuzzy observer (5), (6) is stable if for given positive scalar $\delta \in R$ there exist a symmetric positive definite matrix $P \in R^{n\times n}$ and matrices $S_3 \in R^{n\times n}$, $Y_i \in R^{n\times m}$ such that for all $i = 1, 2, \ldots, s$

$$P = P^T > 0 \tag{19}$$

$$\begin{bmatrix} A_i^T S_3 + S_3^T A_i - Y_i C - C^T Y_i^T & * \\ P - S_3 + \delta S_3^T A_i - \delta Y_{iC} & -\delta(S_3 + S_3^T) < 0 \end{bmatrix} < 0 \tag{20}$$

When the above conditions hold, the set of the observer gain matrices J_i is given by the equations

$$J_i = (S_3^T)^{-1} Y_i \tag{21}$$

Here and hereafter $*$ denotes the symmetric item in a symmetric matrix.

Proof : Since the property of (3) and (12) asserts that

$$\sum_{i=1}^{s} h_i(\theta(t))(A_{ei}e(t) - \dot{e}(t)) = 0 \tag{22}$$

using arbitrary square slack matrices $S_3, S_4 \in R^{n\times n}$ it yields

$$(q^T(t)S_3^T + \dot{q}^T(t)S_4^T)\sum_{i=1}^{s} h_i(\theta(t))(A_{ei}e(t) - \dot{e}(t)) = 0 \tag{23}$$

Adding (23) and transposition of (23) to (15) gives

$$\dot{v}(e(t)) = \dot{e}^T(t)Pe(t) + e^T(t)P\dot{e}(t) + \\ +(e^T(t)S_3^T + \dot{e}^T(t)S_4^T)\sum_{i=1}^{s} h_i(\theta(t))(A_{ei}e(t) - \dot{e}(t)) + \\ + \sum_{i=1}^{s} h_i(\theta(t))(e^T(t)A_{ei}^T - \dot{e}^T(t))(S_3e(t) + S_4\dot{e}(t)) < 0 \tag{24}$$

Then, introducing the notation

$$e^{\circ T}(t) = \begin{bmatrix} e^T(t) & \dot{e}^T(t) \end{bmatrix} \tag{25}$$

after straightforward computation it can be obtained

$$\dot{v}(q(t)) = \sum_{i=1}^{s} \mathrm{h}_i(\theta(t))e^{\circ T}(t)Q_i^{\circ}e^{\circ}(t) < 0 \tag{26}$$

where

$$Q_i^{\circ} = \begin{bmatrix} (A_i - J_iC)^T S_3 + S_3^T(A_i - J_iC) & P - S_3^T + (A_i - J_iC)^T S_4 \\ P - S_3 + S_4^T(A_i - J_iC) & -S_4 - S_4^T \end{bmatrix} < 0 \tag{27}$$

$$S_4 = \delta S_3, \qquad Y_i = S_3^T J_i \tag{28}$$

where $\delta > 0$, $\delta \in R$, then (28) implies (20). This concludes the proof.

The importance of Theorem 1 is that the Lyapunov matrix P is separated from the system matrices A_i, C, i.e. there are no terms containing product of P and any of them. This enables to derive design conditions with respect to natural affine properties of TS models.

It is evident, that Theorem 1 can be simple reformulated considering a symmetric matrix $S_3 = S_3^T$.

4. DESCRIPTOR PRINCIPLE BASED DESIGN METHOD

The results given by Theorem 1 can be generalized using descriptor principle and are formulated as the following theorem.

Theorem 2

The fuzzy observer (22), (23) is stable if for given positive scalar $\delta \in R$ there exist a symmetric positive definite matrix $P_1 \in R^{n \times n}$ and matrices $P_2, P_3 \in R^{n \times n}$, $Y_i \in R^{n \times m}$ such that for all $i = 1, 2, \ldots, s$

$$P_1 = P_1^T > 0, \quad P_2 + P_2^T > 0 \tag{29}$$

$$\begin{bmatrix} A_i^T P_3 + P_3^T A_i - Y_iC - C^T Y_i^T & * \\ P_1 - P_3 + \delta P_3^T A_i - \delta Y_iC & P_2 + P_2^T - \delta(P_3 + P_3^T) \end{bmatrix} < 0 \tag{30}$$

When the above conditions hold, the set of the observer gain matrices J_i is given by the set of the equations

$$J_i = (P_3^T)^{-1} Y_i \tag{31}$$

Proof : Using the equality (22), then with the identities

$$\dot{e}(t) = \dot{e}(t), \qquad \mathbf{0} = \mathbf{0} \tag{32}$$

an equivalent form of (22)can be written as

$$\begin{bmatrix} \boldsymbol{I}_n & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \dot{e}(t) \\ \ddot{e}(t) \end{bmatrix} = \begin{bmatrix} \dot{e}(t) \\ \mathbf{0} \end{bmatrix} = \sum_{i=1}^{s} h_i(\boldsymbol{\theta}(t)) \begin{bmatrix} \mathbf{0} & \boldsymbol{I}_n \\ \boldsymbol{A}_{ei} & -\boldsymbol{I}_n \end{bmatrix} \begin{bmatrix} e(t) \\ \dot{e}(t) \end{bmatrix} \tag{33}$$

or more generally

$$\boldsymbol{E}^{\circ} \dot{e}^{\circ}(t) = \sum_{i=1}^{s} h_i(\boldsymbol{\theta}(t)) \boldsymbol{A}_{ei}^{\circ} e^{\circ}(t) \tag{34}$$

where $e^{\circ}(t)$ is given in (25) and

$$E^{\circ} = E^{\circ T} = \begin{bmatrix} I_n & 0 \\ 0 & 0 \end{bmatrix}, \quad A_{ei}^{\circ} = \begin{bmatrix} 0 & I_n \\ A_{ei} & -I_n \end{bmatrix} \tag{35}$$

Defining the Lyapunov function of the form

$$v(e^{\circ}(t)) = e^{\circ T}(t) E^{\circ T} P^{\circ} e^{\circ}(t) > 0 \tag{36}$$

where

$$E^{\circ T} P^{\circ} = P^{\circ T} E^{\circ} \geq 0 \tag{37}$$

then the derivative of (36) becomes

$$\dot{v}(\boldsymbol{e}^{\circ}(t)) = \dot{e}^{\circ T}(t) \boldsymbol{E}^{\circ T} \boldsymbol{P}^{\circ} \boldsymbol{e}^{\circ}(t) + \boldsymbol{e}^{\circ T}(t) \boldsymbol{P}^{\circ T} \boldsymbol{E}^{\circ} \dot{e}^{\circ}(t) < 0 \tag{38}$$

Inserting (34) in (38) it yields

$$\dot{v}(\boldsymbol{e}^{\circ}(t)) = \boldsymbol{e}^{\circ T}(t) \sum_{i=1}^{s} h_i(\boldsymbol{\theta}(t)) (\boldsymbol{P}^{\circ T} \boldsymbol{A}_{ei}^{\circ} + \boldsymbol{A}_{ei}^{\circ T} \boldsymbol{P}^{\circ}) \boldsymbol{e}^{\circ}(t) < 0 \tag{39}$$

$$P^{\circ T} A_{ei}^{\circ} + A_{ei}^{\circ T} P^{\circ} < 0 \text{ forall } i \tag{40}$$

respectively. Defining

$$P^{\circ} = \begin{bmatrix} P_1 & P_2 \\ P_3 & P_4 \end{bmatrix} \tag{41}$$

then (35), (37) implies

$$P_1 = P_1^T > 0 \tag{42}$$

and using (35) within (13) in (40) it yields

$$\begin{bmatrix} 0 & (A_i - J_i C)^T \\ I_n & -I_n \end{bmatrix} \begin{bmatrix} P_1 & P_2 \\ P_3 & P_4 \end{bmatrix} + \begin{bmatrix} P_1^T & P_3^T \\ P_2^T & P_4^T \end{bmatrix} \begin{bmatrix} 0 & I_n \\ A_i - J_i C & -I_n \end{bmatrix} < 0 \tag{43}$$

After some algebraic manipulations (43) takes the following form

$$\begin{bmatrix} (A_i - J_i C)^T P_3 + P_3^T (A_i - J_i C) & (A_i - J_i C)^T P_4 + P_1^T - P_3^T \\ P_1 - P_3 + P_4^T (A_i - J_i C) & P_2 + P_2^T - P_4 - P_4^T \end{bmatrix} < 0 \tag{44}$$

Setting

$$P_4 = \delta P_3, \qquad Y_i = P_3^T J_i \tag{45}$$

where $\delta > 0$, $\delta \in R$, then (44) implies (30). This concludes the proof.

Remark 1

It is naturally to point out that Theorem 2 is an extension and generalization of Theorem 1, since setting

$$P_2 = 0, \quad P_1 = P, \quad P_3 = S_3 \tag{46}$$

(29), (30) implies (19), (20), respectively. The extension of (30) reflects the Krasovskii theorem property [13] allowing either to consider (24) in the following form

$$\begin{gathered}\dot{v}(e(t)) = \dot{e}^T(t)\boldsymbol{P}e(t) + e^T(t)\boldsymbol{P}\dot{e}(t) + \\ +(e^T(t)\boldsymbol{S}_3^T + \dot{e}^T(t)\boldsymbol{S}_4^T)\sum_{i=1}^{s} h_i(\boldsymbol{\theta}(t))(\boldsymbol{A}_{ei}e(t) - \dot{e}(t)) + \\ +\sum_{i=1}^{s} h_i(\boldsymbol{\theta}(t))(e^T(t)\boldsymbol{A}_{ei}^T - \dot{e}^T(t))(\boldsymbol{S}_3 e(t) + \boldsymbol{S}_4\dot{e}(t)) < -\dot{e}^T(t)(\boldsymbol{S}_2 + \boldsymbol{S}_2^T)\dot{e}(t) < 0\end{gathered} \tag{47}$$

or, equivalently, to define the Lyapunov function in the proof of Theorem 2 as follows

$$v(e(t)) = e^T(t)\boldsymbol{P}e(t) + \int_0^t \dot{e}^T(\tau)(\boldsymbol{S}_2 + \boldsymbol{S}_2^T)\dot{e}(\tau)d\tau > 0 \tag{48}$$

and, as initially, to apply (24) in the proof and, finally, to compare the obtained result with (29), (30) setting

$$P = P_1, \quad S_3 = P_3, \quad S_2 = P_2 \tag{49}$$

Corollary 1

Considering

$$P_2 = 0, \quad P_4 = 0, \quad P_1 = P_3 \tag{50}$$

then (44) reduces to

$$\begin{bmatrix} (A_i - J_iC)^T P_1 + P_1^T(A_i - J_iC) & 0 \\ 0 & 0 \end{bmatrix} \le 0 \tag{51}$$

which implies

$$(A_i - J_iC)^T P_1 + P_1^T(A_i - J_iC) < 0 \tag{52}$$

It is obvious that with

$$P = P_1 = P_1^T, \quad Y_i = P_1^T J_i = PJ_i \tag{53}$$

(52) implies (8).

These modifications give the possibility to achieve the degree of conservatism that is most appropriate for a TS system.

5. ILLUSTRATIVE EXAMPLE

The considered system is represented by the TS fuzzy model (1), (2) with s = 3 and the system model parameters

$$A_1 = \begin{bmatrix} -1.0522 & -1.8666 & 0.5102 \\ -0.4380 & -5.4335 & 0.9205 \\ -0.5522 & 0.1334 & -0.4898 \end{bmatrix}, A_2 = \begin{bmatrix} -1.0565 & -1.8661 & 0.5116 \\ -0.4380 & -5.4359 & 0.9214 \\ -0.5565 & 0.1339 & -0.4884 \end{bmatrix}$$

$$A_3 = \begin{bmatrix} -1.0602 & -1.8657 & 0.5133 \\ -0.4381 & -5.4353 & 0.9216 \\ -0.5602 & 0.1343 & -0.4867 \end{bmatrix}, B = \begin{bmatrix} 3 & 1 \\ 1 & -1 \\ 3 & 0 \end{bmatrix}, C^T = \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 0 & 1 \end{bmatrix}$$

where matrices B_i are the same for all i and the premise variable and the membership functions for approximation of $f(q_1(t))$ in the prescribed sector are given as

$$\theta(t) = \begin{bmatrix} \theta_1(t) \\ \theta_2(t) \\ \theta_3(t) \end{bmatrix}, \theta_i = \begin{cases} \theta_1(t) \text{ if } q_1(t) \text{ is about } 5, \\ \{\theta_2(t) \text{ if } q_1(t) \text{ is about } 0, \\ \theta_3(t) \text{ if } q_1(t) \text{ is about } -5, \end{cases}$$

$$h_1(\theta_2(t)) = 1 - \frac{1}{5} | \theta_2(t) - 5 |, \quad h_2(\theta_1(t)) = 1 - \frac{1}{5} | \theta_1(t) |, \quad h_3(\theta_3(t)) = 1 - \frac{1}{5} | \theta_3(t) + 5 |$$

Solving the variables P, Y_i, $i = 1,2,3$ satisfying (7), (8) via the LMI technique using toolbox SeDuMi [14] gave the following results

$$P = \begin{bmatrix} 0.6353 & -0.1200 & -0.0377 \\ -0.1200 & 0.2368 & 0.0521 \\ -0.0377 & 0.0521 & 0.6776 \end{bmatrix}$$

$$Y_1 = \begin{bmatrix} -0.0969 & -0.1124 \\ -0.3185 & -0.1973 \\ 0.0101 & 0.1690 \end{bmatrix}, Y_2 = \begin{bmatrix} -0.0992 & -0.1120 \\ -0.3173 & -0.1979 \\ 0.0089 & 0.1698 \end{bmatrix}, Y_3 = \begin{bmatrix} -0.1002 & -0.1118 \\ -0.3170 & -0.1977 \\ 0.0082 & 0.1709 \end{bmatrix}$$

and, besides, the fuzzy observer gain matrices were obtained as follows

$$J_1 = \begin{bmatrix} -0.4474 & -0.3634 \\ -1.5966 & -1.0859 \\ 0.1126 & 0.3127 \end{bmatrix}, J_2 = \begin{bmatrix} -0.4502 & -0.3632 \\ -1.5925 & -1.0888 \\ 0.1104 & 0.3140 \end{bmatrix}, J_3 = \begin{bmatrix} -0.4517 & -0.3627 \\ -1.5914 & -1.0881 \\ 0.1092 & 0.3157 \end{bmatrix}$$

guaranteeing the stable eigenvalues spectra of the local observer system matrices in such a way that

$$\rho(A_{e1}) = \{-0.7560, -1.7011 \pm 1.1316\mathrm{i}\}, \quad \rho(A_{e2}) = \{-0.7575, -1.7029 \pm 1.1269\mathrm{i}\}$$

$$\rho(A_{e3}) = \{-0.7599, -1.7034 \pm 1.1265\mathrm{i}\}$$

Applying the same toolbox to solve LMIs (19), (20) conditioned by $\delta = 1$, the obtained set of matrix variables was as follows

$$P=\begin{bmatrix} 0.6415 & -0.1152 & 0.0157 \\ -0.1152 & 0.7158 & -0.0994 \\ 0.0157 & -0.0994 & 0.7004 \end{bmatrix}, S_3=\begin{bmatrix} 0.3577 & -0.0386 & -0.0348 \\ -0.1063 & 0.1468 & 0.0100 \\ 0.0595 & 0.0316 & 0.3587 \end{bmatrix}$$

$$Y_1=\begin{bmatrix} -0.0188 & 0.0224 \\ -0.1142 & -0.0324 \\ -0.1452 & 0.1856 \end{bmatrix}, Y_2=\begin{bmatrix} -0.0191 & 0.0248 \\ -0.1155 & -0.0323 \\ -0.1465 & 0.1828 \end{bmatrix}, Y_3=\begin{bmatrix} -0.0227 & 0.0237 \\ -0.1138 & -0.0327 \\ -0.1458 & 0.1833 \end{bmatrix}$$

so that the local observers gain matrices were given as

$$J_1=\begin{bmatrix} -0.2067 & -0.1318 \\ -0.7449 & -0.3663 \\ -0.4039 & 0.5149 \end{bmatrix}, J_2=\begin{bmatrix} -0.2097 & -0.1226 \\ -0.7543 & -0.3616 \\ -0.4076 & 0.5079 \end{bmatrix}, J_3=\begin{bmatrix} -0.2171 & -0.1270 \\ -0.7449 & -0.3652 \\ -0.4067 & 0.5088 \end{bmatrix}$$

This set of gains embedded the eigenvalues spectra of the local observer system matrices as follows

$$\rho(A_{e1})=\{-4.1633, -1.0047\pm 0.0879\mathrm{i}\}, \quad \rho(A_{e2})=\{-4.1600, -1.0016\pm 0.0804\mathrm{i}\}$$

$$\rho(A_{e3})=\{-4.1703, -0.9967\pm 0.0917\mathrm{i}\}$$

Finally, solving LMIs (29), (30) conditioned by $\delta=1$, a feasible solution produced the following LMI variables

$$P_1=\begin{bmatrix} 1.0236 & -0.2562 & 0.0263 \\ -0.2562 & 1.0633 & -0.1593 \\ 0.0263 & -0.1593 & 1.1439 \end{bmatrix}, P_3=\begin{bmatrix} 0.7114 & -0.1530 & 0.1444 \\ -0.1842 & 0.2562 & -0.0228 \\ -0.1177 & 0.1345 & 0.7273 \end{bmatrix}$$

$$Y_1=\begin{bmatrix} -0.1798 & 0.0085 \\ -0.1937 & -0.0745 \\ -0.3158 & 0.1619 \end{bmatrix}, Y_2=\begin{bmatrix} -0.1816 & 0.0096 \\ -0.1938 & -0.0745 \\ -0.3186 & 0.1637 \end{bmatrix}, Y_3=\begin{bmatrix} -0.1833 & 0.0106 \\ -0.1937 & -0.0744 \\ -0.3210 & 0.1655 \end{bmatrix}$$

where, for simplicity, P_2 is not listed. This result provides TS fuzzy state observer with the following local gain matrices

$$J_1=\begin{bmatrix} -0.5430 & -0.0672 \\ -0.8942 & -0.4476 \\ -0.3544 & 0.2220 \end{bmatrix}, J_2=\begin{bmatrix} -0.5462 & -0.0653 \\ -0.8950 & -0.4475 \\ -0.3576 & 0.2241 \end{bmatrix}, J_3=\begin{bmatrix} -0.5491 & -0.0635 \\ -0.8951 & -0.4470 \\ -0.3604 & 0.2262 \end{bmatrix}$$

and with the eigenvalues spectra of the local observer system matrices

$$\rho(A_{e1})=\{-4.0034, -0.6547\pm 0.1088\mathrm{i}\}, \quad \rho(A_{e2})=\{-4.0056, -0.6553\pm 0.1090\mathrm{i}\}$$

$$\rho(A_{e3})=\{-4.0060, -0.6557\pm 0.1095\mathrm{i}\}$$

Applying the designed to the TS fuzzy system model with the initial condition

$$q_e^T(0)=0, \quad u^T(t)=0, \quad q^T(0)=\begin{bmatrix} 0.3 & 0.6 & 0.9 \end{bmatrix}$$

the simulation results are stated in Fig. 1 to Fig. 3 to illustrate the estimated output behaviour of the system sequentially as the observers parameters were computed using Lemma 1, Theorem 1 and Theorem 2. It is evident that the best compromise in the settling time and overshooting gives the result of Theorem 2.

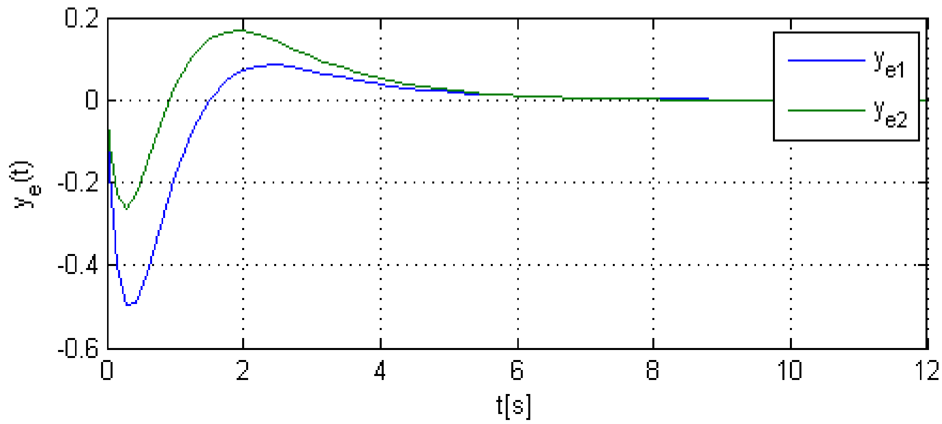

Figure 1: TS fuzzy observer output variables response (based on Lemma 1 results)

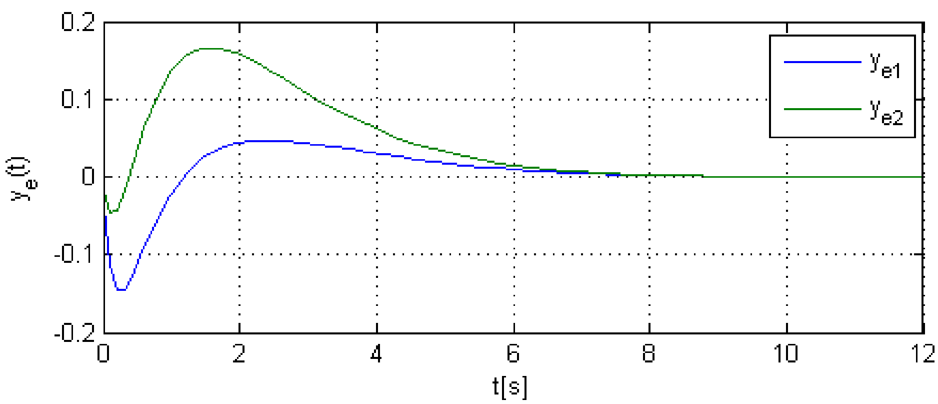

Figure 2: TS fuzzy observer output variables response (based on Theorem 1 results)

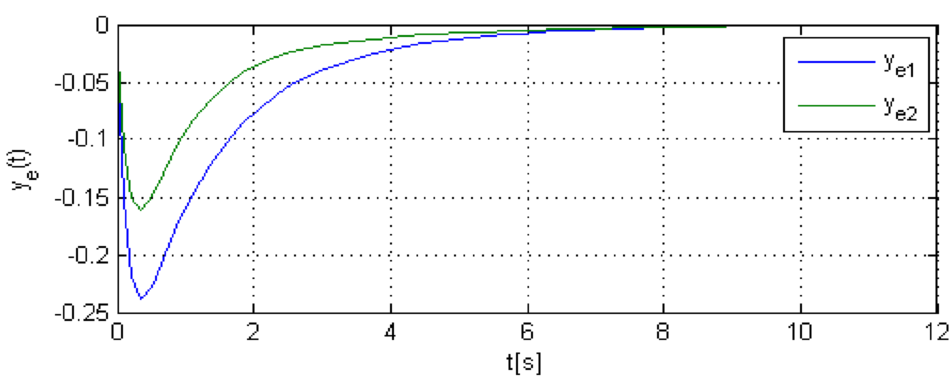

Figure 3: TS fuzzy observer output variables response (based on Lemma 2 results)

6. CONCLUDING REMARKS

New approach for output dynamic feedback control design is presented in this paper. By the proposed procedure the control problem is parameterized in such LMIs set with one additional LME which admit more freedom in guaranteeing the output feedback control performance for a bi-proper dynamic controller and by LMIs set only for a strictly proper dynamic output controller. Sufficient conditions of the controller existence manipulating the stability of the closed-loop systems imply the control structure, which stabilize the system in the sense of Lyapunov and the controller design tasks is a solvable numerical problem. An additional benefit of the method is that controller uses minimum feedback information with respect to desired system output and the approach is enough flexible to allow the inclusion of additional design condition bounds.

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