ENTROPY-COST RATIO MAXIMIZATION MODEL FOR EFFICIENT STOCK PORTFOLIO SELECTION USING INTERVAL ANALYSIS

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ABSTRACT

This paper introduces a new stock portfolio selection model in non-stochastic environment. Following the principle of maximum entropy, a new entropy-cost ratio function is introduced as the objective function. The uncertain returns, risks and dividends of the securities are considered as interval numbers. Along with the objective function, eight different types of constraints are used in the model to convert it into a pragmatic one. Three different models have been proposed by defining the future financial market optimistically, pessimistically and in the combined form to model the portfolio selection problem. To illustrate the effectiveness and tractability of the proposed models, these are tested on a set of data from Bombay Stock Exchange (BSE). The solution has been done by genetic algorithm.

KEYWORDS

Stock Portfolio Selection, Interval Numbers, Entropy-Cost Ratio model, Dividend, Genetic Algorithm.

1. INTRODUCTION

Louis Bachelier's [1] dissertation theory on speculation in 1900 is probably the first attempt to the modern mathematical models of finance. In 1952, Markowitz [2] prominently starts the evolution in modern portfolio selection model analysis and finance management era. Many reasonable works were done since then in this area; the common attempt was maximizing the return or minimizing the risk. Allocation of capital fraction in different risky assets in the most efficient way was always the goal of the researchers.

Many interesting works have been done using different concepts and constraints. Some reasonable works were done using the first two moments of return distributions. Some researchers also have used higher order moments. Arditti [3], Samuelson [4], Kraus and Litzenberger [5], Konno *et al.* [6], Konno and Suzuki [7], Liu *et al.* [8], Prakash *et al.* [9], Lai [10], Chunhachinda *et al.* [11], Briec *et al.* [12], Yu *et al.* [13] are some among those researchers who have invoked these concepts in this area.

However, most of them assumed the security returns as random variables. In 1965, one of the innovative theories of mathematics, Fuzzy set theory, is proposed by Zadeh [14]. Realizing the limitations of stochastic approach to handle such a non-stochastic factor (risky assets), researchers start to involve fuzzy approach to portfolio selection models formulation. Ramaswamy [15], Parra *et al.* [16], Zhang and Nie [17], Bilbao-Terol *et al.* [18], Gupta *et al.* [19], Huang [20, 21], Lin and Liu [22], Bhattacharyya *et al.* [23, 24, 25], Li *et al.* [26] and others study fuzzy portfolio selection models.

A special approach, based on interval approximation of fuzzy numbers, assumes that the information of a decision making problem are not well defined but may vary in given intervals. Interval valued fuzzy set theory is introduced separately in the mid-seventies by Grattan-Guinness [27], Jahn [28], Sambuc [29] and Zadeh [30]. Many researchers use interval programming to portfolio selection problem. Parra *et al.* [16] develop a goal programming (GP) model for portfolio selection, based on the expected intervals of fuzzy numbers that define the objectives and target values. Lai *et al.* [31] extend the Markowitz's model to an interval programming model by quantifying the expected return and the covariance as intervals. Ida [32, 33], Giove *et al.* [34], Fang *et al.* [35], Ehrgott *et al.* [36], Bhattacharya *et al.* [37] use interval programming model in various ways. Dividend, transaction cost, liquidity, short and long term return have been used in these models along with mean, variance, skewness.

Maximization entropy principle and minimization cross-entropy principle motivated Kapur and Hesavan [38] to propose an entropy maximization model in 1992. The principle of maximum entropy states that, subject to precisely stated prior data like a proposition that expresses testable information, the probability distribution which best represents the current state of knowledge is the one with largest information-theoretical entropy. The principle was first expounded by E.T. Jaynes in two papers [39, 40] in 1957, where he emphasized a natural correspondence between statistical mechanics and information theory. Later, Fang *et al.* [41], Qin *et al.* [42], Li *et al.* [43], Liu [44] and others also investigate entropy optimization models, which are widely accepted.

The purpose of this paper is to build a new portfolio selection model by making a bridge between these two concepts. Here we have proposed a new objective function as a ratio of entropy to total transaction cost which we want to maximize under the usual constraints on return, risk, dividend etc., but the constraints are treated here in fuzzy interval analytic way. The well known theory of interval numbers and the reason to use it in portfolio models are discussed in the following two sections. In next sections the objective function and constraints are proposed. Then the interval model and an algorithm to solve the problem are discussed. The algorithm is based on the well known Genetic Algorithm; some improvisations have been done to make the algorithm more effective for the particular models. Finally real life market data extracted from Bombay Stock Exchange (BSE) are used to solve the stated models to prove the efficiency of this new approach.

2. FUZZY INTERVAL ANALYSIS

Let R be the set of all real numbers. An order pair in a bracket defines an interval

$$A = [\underline{\alpha}, \ \bar{\alpha}] = \{x : \underline{\alpha} \le x \le \bar{\alpha}, x \in R\},$$

where $\underline{\alpha}$ is lower bound and $\bar{\alpha}$ is upper bound of interval A. The centre and the width of A are defined as,

$$m(A) = \frac{\bar{\alpha} + \underline{\alpha}}{2} \ \text{and} \ w(A) = \frac{\bar{\alpha} - \underline{\alpha}}{2}.$$

A can also be denoted by its centre and width as

$$A = \langle m(A), w(A) \rangle = \{x : m(A) - w(A) \leq x \leq m(A) + w(A), x \in R\}.$$

The annex of ordinary arithmetic to closed intervals is recognized as interval arithmetic. First, we extract some fundamental concepts, as follows:

Definition 1. Let $o \in \{+,-,\times,\div\}$ be a binary operation on R. Let A and B be two closed intervals. Then the binary operation on the set of all closed intervals is defined by,

$$AoB = \{xoy : x \in A, y \in B\}.$$

In the case of division, it is always assumed that $0 \notin B$. The operations on the intervals are as follows:

$$A + B = \left[\underline{\alpha} + \underline{\beta}, \overline{\alpha} + \overline{\beta}\right],$$

$$A - B = \left[\underline{\alpha} - \underline{\beta}, \overline{\alpha} - \overline{\beta}\right],$$

$$A \pm k = \left[\underline{\alpha} \pm k, \ \overline{\alpha} \pm k\right], k \in R$$

$$kA = k\left[\underline{\alpha}, \overline{\alpha}\right] = \begin{cases} \left[k\underline{\alpha}, k\overline{\alpha}\right], k \geq 0 \\ \left[k\overline{\alpha}, k\underline{\alpha}\right], k \leq 0 \end{cases}, k \in R$$

An interval number is considered as a special fuzzy number whose membership function takes the value 1 over the interval, and 0 anywhere else. Clearly, the above four operations of the intervals are equivalent to the operations of addition, subtraction and scalar multiplication of fuzzy numbers by means of the extension principle of Zadeh [45]. Rommelfanger *et al.* [46] inspect the interval programming problem like a fuzzy programming problem. Ishibuchi and Tanaka [47] suggest an order relation among two intervals.

Definition 2. For any two interval numbers $A = \left[\underline{\alpha}, \overline{\alpha}\right]$ and $B = \left[\underline{\beta}, \overline{\beta}\right]$, there is an interval inequality relation $A \leq B$ among the two interval numbers A and B if and only if, $m(A) \leq m(B)$. .Furthermore, if $\overline{\alpha} \leq \underline{\beta}$, the interval inequality relation $A \leq B$ between A and B is said to be optimistic satisfactory; if $\overline{\alpha} \geq \underline{\beta}$, the interval inequality relation $A \leq B$ between A and B is said to be pessimistic satisfactory.

3. USING INTERVAL ANALYSIS IN PORTFOLIO MODELLING

We know that, in any emerging market, future returns of securities cannot be accurately predicted. Generally, researchers consider the arithmetic mean of historical returns as the expected return of the security. Thus the expected return of the security is a crisp value. Two main problems arise here:

Firstly, if the time horizon of the historical data of a security is very long, the influence of the recent data becomes negligible over that of the historical data.

Secondly, if the historical data of a security is not available in a large scale, one cannot accurately estimate the statistical parameters, due to data scarcity.

Considering these two problems, it is better to consider the expected return of a security as an interval number, rather than a crisp value, based on the arithmetic mean of historical data. Financial reports and the security's historical data can be used to determine the expected return

interval's range. To determine the range of change in expected returns of securities, the following three factors can be considered:

Arithmetic mean: Arithmetic means of returns of securities should not be expressed as expected returns directly, but can be used as a good approximation. Denote the arithmetic mean return factor as $A(r_i)$, which can be calculated with historical data.

Historical return tendency: If recent returns of a security have been increasing, the expected return of the security is greater than the arithmetic mean based on historical data. However, if recent returns of a security have been declining, the expected return of the security is smaller than the arithmetic mean based on historical data. Denote the historical return tendency factor as $H(r_i)$, which reflects the tendency of the return on the security. We can use the arithmetic mean of recent returns as $H(r_i)$
.

Forecast of future returns of a security: The third factor influencing the expected return of a security is its estimated future returns. In a risky modern market scenario the expected return is highly influenced by the present financial condition of a particular corporation which may be highly anomalous. Denote the forecast return factor as $F(r_i)$. Computation of derivation of $F(r_i)$ requires some forecasts based on the financial reports and individual experiences of experts of the market.

Based on the above three factors, we can derive lower and upper limits of the expected return of the security. We can put the minimum of the three factors $A(r_i)$, $H(r_i)$ and $F(r_i)$ as the lower limit of the expected return, while we can put the maximum value of the three factors as the upper limit of the expected return of the security.

Similarly, in fuzzy environment the risk and dividend are also unpredictable. So the variances, covariance and dividends are also considered here as fuzzy interval numbers.

4. THE PORTFOLIO SELECTION MODEL

Here we first describe the assumptions and notations used in this paper. Then in the subsequent subsections the objective function and the constraints have been constructed. The fourth subsection presents three different mathematical models for different situations. Finally the optimization algorithm, which is used here to solve the models, has been discussed.

4.1 Assumptions and notations

Let us consider a financial market with n risky assets. An investor wants to allocate his wealth among these risky assets. For i^{th} risky asset $(i = 1, 2, 3, ..., n)$, let us use the following notations:

x_i = the proportion of the total capital to be invested,

d_i = the estimated annual dividend in the next year,

r_i = the expected return,

$\sigma_{ij} = \text{cov}(r_i, r_j)$, the covariance between r_i and r_j,

c_i = the transaction cost,

k_i = the constant cost per change in a proportion, $k_i \geq 0$.

4.2   Formulation of objective function

The Principle of Maximum Entropy: Entropy is used to measure the uncertainty associated with each random variable. The Principle of Maximum Entropy is based on the premise that when estimating the probability distribution, you should select that distribution which leaves you the largest remaining uncertainty (i.e., the maximum entropy) consistent with your constraints. That way you have not introduced any additional assumptions or biases into your calculations.

Following the spirit of maximum entropy principle, the total entropy of the allocated proportions of investor's wealth, $E = -\sum_{i=1}^{n} x_i \ln x_i$ is introduced in the objective function. Kapur [38] introduces it earlier as a single term in that objective function.

Total transaction cost: Let me consider the transaction cost c_i to be a V-shaped function of the difference between a given portfolio $x^0 = \left(x_1^0, x_2^0, x_3^0, ..., x_n^0 \right)$ and a new portfolio $x = \left(x_1, x_2, x_3, ..., x_n \right)$..Hence, the total transaction cost is,

$$C = \sum_{i=1}^{n} c_i = \sum_{i=1}^{n} k_i \left| x_i - x_i^0 \right|,$$

which we want to minimize.

Taking these two aspects in consideration, the objective function is formulated as:

$$\max \frac{-\sum_{i=1}^{n} x_i \ln x_i}{\sum_{i=1}^{n} k_i \left| x_i - x_i^0 \right|} = \max \frac{E}{C}.$$

4.3 Construction of the constraints with interval coefficients

The expected return of portfolio $x = \left(x_1, x_2, x_3, ..., x_n \right)$ with transaction cost is given by,

$$R(x) = \sum_{i=1}^{n} r_i x_i - \sum_{i=1}^{n} k_i \left| x_i - x_i^0 \right|.$$

Variance of the portfolio is,

$$\sigma^2(x) = \sum_{i=1}^{n} \sum_{j=1}^{n} \sigma_{ij} x_i x_j,$$

And the estimated annual dividend of the portfolio is,

$$D(x) = \sum_{i=1}^{n} d_i x_i.$$

For a new investor it can be assumed that $x_i^0 = 0, \forall i = 1, 2, 3, ..., n.$

From our previous discussion on interval analysis, let $\Delta r_{il} = r_i - \min \left\{ A(r_i), H(r_i), F(r_i) \right\}$ and $\Delta r_{ig} = \max \left\{ A(r_i), H(r_i), F(r_i) \right\} - r_i$..

Then the fuzzy expected return for the ith security is represented by the interval number:

$$\tilde{r}_i = \left[r_i - \Delta r_{il}, r_i + \Delta r_{ig} \right].$$

Similarly, σ_{ij} and d_i, respectively are represented by:

$$\tilde{\sigma}_{ij} = \left[\sigma_{ij} - \Delta \sigma_{ijl}, \sigma_{ij} + \Delta \sigma_{ijg} \right], \ \tilde{d}_i = \left[d_i - \Delta d_{il}, d_i + \Delta d_{ig} \right].$$

Then the fuzzy expected return, risk and dividend are respectively defined by,

$$\tilde{R}(x) = \sum_{i=1}^{n} \tilde{r}_i x_i - \sum_{i=1}^{n} k_i \left| x_i - x_i^0 \right|,$$

$$\tilde{\sigma}^2(x) = \sum_{i=1}^{n} \sum_{j=1}^{n} \tilde{\sigma}_{ij} x_i x_j,$$

$$\tilde{D}(x) = \sum_{i=1}^{n} \tilde{d}_i x_i.$$

Since $x_i \geq 0$, we have,

$$\tilde{R}(x) = \left[R(x) - \Delta_{Rl}(x), R(x) + \Delta_{Rg}(x) \right],$$

$$\tilde{\sigma}^2(x) = \left[\sigma^2(x) - \Delta_{Vl}(x), \sigma^2(x) + \Delta_{Vg}(x) \right],$$

$$\tilde{D}(x) = \left[D(x) - \Delta_{Dl}(x), D(x) + \Delta_{Dg}(x) \right],$$

where,

$$R(x) - \Delta_{Rl}(x) = \sum_{i=1}^{n} (r_i - \Delta r_{il}) x_i - \sum_{i=1}^{n} k_i \left| x_i - x_i^0 \right|,$$

$$R(x) + \Delta_{Rg}(x) = \sum_{i=1}^{n} (r_i + \Delta r_{ig}) x_i - \sum_{i=1}^{n} k_i \left| x_i - x_i^0 \right|;$$

$$\sigma^2(x) - \Delta_{Vl}(x) = \sum_{i=1}^{n} \sum_{j=1}^{n} (\sigma_{ij} - \Delta \sigma_{ijl}) x_i x_j,$$

$$\sigma^2(x) + \Delta_{Vg}(x) = \sum_{i=1}^{n} \sum_{j=1}^{n} (\sigma_{ij} + \Delta \sigma_{ijg}) x_i x_j;$$

$$D(x) - \Delta_{Dl}(x) = \sum_{i=1}^{n} (d_i - \Delta d_{il}) x_i,$$

$$D(x) + \Delta_{Dg}(x) = \sum_{i=1}^{n} (d_i + \Delta d_{ig}) x_i.$$

Now, an investor always prefers a portfolio with having a minimum expected return, a maximum risk tolerance and a higher dividend. So, the following constraints are proposed:

$$\tilde{R}(x) \geq \alpha \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots C1$$

$$\tilde{\sigma}^2(x) \leq \beta \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots C2$$

$$\tilde{D}(x) \geq \gamma \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots C3$$

Here α, β, γ will be allocated by the investor according to the preference.

The well known capital budget constraint on the assets is presented by:

$$\sum_{i=1}^{n} x_i = 1 \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots C4$$

The maximum and minimum fractions of the capital budget being allocated to each of the assets in the portfolio depend upon factors like price relative to the asset in comparison with the average of the price of all the assets in the chosen portfolio, minimal lot size that can be traded in the market, the past performance of the price of the asset, information available about the issuer of the asset, trends in the business of which it is a division etc. Different investors having different views may allocate the same overall capital budget differently.

Let the maximum fraction of the capital that can be invested in a single asset i is M. Then

$$x_i \leq M , (\forall i = 1, 2, 3, \dots\dots\dots, n) \dots\dots\dots\dots C5$$

Let the minimum fraction of the capital that can be invested in a single asset i is m. Then

$$x_i \geq m, (\forall i = 1, 2, 3, \dots\dots\dots, n) \dots\dots\dots\dots C6$$

The above two constraints ensure that neither a huge amount nor a very tiny amount of the assets are assigned in a single stock of the portfolio. Huge amount of investment of the asset in a single stock opposes the motto of selecting a portfolio (i.e., diversification of investment). On the other hand, negligible amount of investment in a portfolio is impractical. For example, investing neither 80% nor 0.5% of the asset in a single stock of the portfolio is good. Note that for $(n-k)$ number of stocks, we have $x_i = 0$.

No short selling is considered in the portfolio here. So we have,

$$x_i \geq 0, (\forall i = 1, 2, 3, \dots\dots\dots, n) \dots\dots\dots\dots C7$$

Moreover, it is very difficult for an investor to maintain capital allocations for all available assets in the market, so one can decide to deal with any k no. of assets out of a total of n assets. This leads to the following constraint:

$$\sum_{i=1}^{n} sign(x_i) = k \dots\dots\dots\dots\dots\dots\dots\dots\dots C8 ,$$

Here $sign(x_i)$ represents the signum functional value of x_i.

These are the eight main constraints to be considered in the model discussed in this paper. Obviously, different investors have different approaches towards a problem.

4.4 Construction of the portfolio selection models

Using the objective function and the constraints obtained in the previous subsections, the following fuzzy investment problem is constructed:

$$\langle 1 \rangle \begin{cases} max \dfrac{E}{C} \\ subject\ to \\ \tilde{R}(x) \geq \alpha,\ \tilde{\sigma}^2(x) \leq \beta,\ \tilde{D}(x) \geq \gamma, \sum\limits_{i=1}^{n} sign(x_i) = k \\ \sum\limits_{i=1}^{n} x_i = 1,\ x_i \leq M,\ x_i \geq m,\ x_i \geq 0, (\ \forall i = 1,2,3,................,n\ ). \end{cases}$$

Now, denote:

$$R_l(x) = R(x) - \Delta_{Rl}(x), R_g(x) = R(x) + \Delta_{Rg}(x)$$
$$\sigma^2{}_l(x) = \sigma^2(x) - \Delta_{Vl}(x), \sigma^2{}_g(x) = \sigma^2(x) + \Delta_{Vg}(x)$$
$$D_l(x) = D(x) - \Delta_{Dl}(x), D_g(x) = D(x) + \Delta_{Dg}(x)$$

Then, clearly,

$$\tilde{R}(x) = \left[R_l(x), R_g(x) \right],\ \tilde{\sigma}^2(x) = \left[\sigma^2{}_l(x), \sigma^2{}_g(x) \right],\ \tilde{D}(x) = \left[D_l(x), D_g(x) \right].$$

From the fuzzy interval optimization model $\langle 1 \rangle$, let us construct the following three crisp optimization models ($\langle 2 \rangle, \langle 3 \rangle, \langle 4 \rangle$), with the help of interval programming by Lai *et al.* [31].

In the following model $\langle 2 \rangle$, the investor estimates the return, risk and dividend pessimistically and aims to optimize the objective based on such constraints.

$$\langle 2 \rangle \begin{cases} max \dfrac{E}{C} \\ subject\ to \\ R_g(x) \geq \alpha,\ \sigma^2{}_l(x) \leq \beta,\ D_g(x) \geq \gamma, \sum\limits_{i=1}^{n} sign(x_i) = k \\ \sum\limits_{i=1}^{n} x_i = 1,\ x_i \leq M,\ x_i \geq m,\ x_i \geq 0, (\forall i = 1,2,3,................,n). \end{cases}$$

In the following model $\langle 3 \rangle$, the investor estimates the return, risk and dividend optimistically and aims to optimize the objective based on such constraints.

$$\langle 3 \rangle \begin{cases} max \dfrac{E}{C} \\ subject\ to \\ R_l(x) \geq \alpha,\ \sigma^2{}_g(x) \leq \beta,\ D_l(x) \geq \gamma, \sum\limits_{i=1}^{n} sign(x_i) = k \\ \sum\limits_{i=1}^{n} x_i = 1, x_i \leq M,\ x_i \geq m,\ x_i \geq 0, (\forall i = 1,2,3,................,n). \end{cases}$$

In following the model $\langle 4 \rangle$, the investor chooses a balanced one; to some extent she/he is optimistic, but to some extent she/he is pessimistic too.

$$\langle 4 \rangle \begin{cases} \max \dfrac{E}{C} \\ subject\ to \\ \lambda_1 R_l(x) + (1-\lambda_1) R_g(x) \geq \alpha,\ \lambda_2 \sigma^2_g(x) + (1-\lambda_2)\sigma^2_l(x) \leq \beta, \\ \lambda_3 D_l(x) + (1-\lambda_3) D_g(x) \geq \gamma,\ \sum_{i=1}^{n} x_i = 1, \\ x_i \leq M,\ x_i \geq m,\ x_i \geq 0, (\forall i = 1,2,3,................,n), \\ \sum_{i=1}^{n} sign(x_i) = k. \end{cases}$$

Here, $\lambda_j \in (0,1)$ $[j = 1,\ 2,\ 3]$ would be assigned by the investor.

4.5 The optimization algorithm

Genetic algorithm (GA) is a stochastic search method used in computer science field of artificial intelligence and is based on the principles of natural genetic systems. It performs a multidimensional search in providing an optimal solution for evaluation function of an optimization problem mimicking the process of natural evolution. Genetic algorithms belong to the larger class of evolutionary algorithms (EA), which generate solutions to optimization problems using techniques inspired by natural evolution, such as inheritance, mutation, selection, and crossover. Since Holland [48] first proposed it in 1975, genetic algorithm has been widely studied, experimented and applied in many fields like operations research, finance, industrial engineering, VLSI design, pattern recognition, image processing etc.

While solving an optimization problem using genetic algorithm, a population of solutions is evolved towards better solutions. Each population has a set of individuals, its chromosomes or genotypes, which can be mutated and altered. The evolution usually starts from a population of randomly generated individuals and are changed in future generations. In each generation, the fitness of every individual in the population is evaluated, the more fit individuals are stochastically selected from the current population, and each individual's genome is modified using three basic operations on the individuals of the population. The operations are selection, crossing over and mutation. The new population obtained after selection, cross over and mutation is then to generate another population. The new population is then used in the next iteration of the algorithm. Commonly, the algorithm terminates when either a maximum number of generations has been produced, or a satisfactory fitness level has been reached for the population. If the knowledge about the best string is preserved within the population, such a model is called a genetic algorithm with an elitist model (EGA). An EGA converges to the global solution with any choice of initial population (c.f. Bhandari et al. [49]). To find the optimal portfolio, we integrate fuzzy simulation into the GA. The GA procedure has been introduced in detail in Huang [50]. Here, we summarize the algorithm as follows:

1. In the GA, a solution $x = (x_1, x_2, x_3,................,x_n)$ is represented by the chromosome:

$C = (c_1, c_2, c_3, ..., c_n)$, where the genes $c_1, c_2, c_3, ..., c_n$ are in the interval [0, 1]. The matching between the solution and the chromosome is through

$x_i = c_i / (c_1 + c_2 + c_3 + ... + c_n), i = 1, 2, 3, ..., n$, which ensures that,

$x_1 + x_2 + x_3 + ... + x_n = 1$ always holds.

Randomly generate a point C. Use fuzzy simulation to calculate the values of return, risk and dividend to check the feasibility of the chromosomes. Take the feasible chromosomes as the initial population.

2. Calculate the objective values for all chromosomes by fuzzy simulation. Then, select the best among the chromosomes according to the objective values. The chromosome having the maximum z-value is kept in memory by continuous update. Next, pass the population to perform crossover and mutation.

3. Update the chromosomes by crossover and mutation operations. Check the feasibility of the chromosomes to build a feasible new population.

4. Once again calculate the objective values for all chromosomes by fuzzy simulation. Select the best among the chromosomes according to the objective values. The chromosome having the maximum z-value is kept in memory by continuous update. Next, again pass the population to perform crossover and mutation.

5. Repeat steps 2 to 4 a number of times.

6. The best chromosome obtained at the last iteration is the required solution.

5. CASE STUDY: APPLICATION TO BOMBAY STOCK EXCHANGE (BSE)

Now the portfolio selection model discussed above will be applied to the stock market of Bombay stock exchange (BSE).   Bombay Stock Exchange is a stock exchange located on Dalal Street, Mumbai, Maharashtra, India. It is the 10th largest stock exchanges in the world by market capitalization. Established in 1875, BSE Ltd. (formerly known as Bombay Stock Exchange Ltd.), is Asia's first Stock Exchange and one of India's leading exchange groups. Over the past 137 years, BSE has facilitated the growth of the Indian corporate sector by providing it an efficient capital-raising platform. Popularly known as BSE, the bourse was established as "The Native Share & Stock Brokers' Association" in 1875. Around 5000 companies are listed on BSE making it world's No. 1 exchange in terms of listed members. The companies listed on BSE Ltd command a total market capitalization of USD Trillion 1.2 as of October 31, 2012. BSE Ltd is world's fifth most active exchange in terms of number of transactions handled through its electronic trading system. It is also one of the world's leading exchanges (3rd largest in July 2012) for Index options trading (Source: World Federation of Exchanges).

Here, we have taken 10 companies which are included in BSE index. Their return, dividend and other history has been collected and analyzed carefully for January, 2005 to March, 2010. The data collected is shown in Table 1. The covariance matrix of the return rates of these risky assets are shown in tables 2 and 3.

Table 1. Data of Return & Dividend

COMPANY	HIGHEST RETURN	LOWEST RETURN	HIGHEST DIVIDEND	LOWEST DIVIDEND
TATA POWER	1.2145	-0.28425	1.20	0.85
RELIANCE INFRA	1.78775	-0.484	0.70	0.11
ITC	0.49725	-0.3775	3.70	2.65
M&M	1.135	-0.3915	1.30	0.40
GRASIM	0.9465	-0.27625	3.00	1.40
SBI	0.9775	-0.25225	2.90	1.00
WIPRO	0.78025	-0.43975	2.50	0.50
TATA STEEL	1.3065	-0.446	1.60	1.00
ONGC	0.62975	-0.3605	2.50	1.30
TATA MOTORS	1.3885	-0.504	1.50	0.40

Table 2. Lower Covariance Matrix

STOCK	TATA POWER	RELIANCE INFRA	ITC	M&M	GRASIM	SBI	WIPRO	TATA STEEL	ONGC	TATA MOTORS
TATA POWER	0.038665	0.048035	-0.04569	0.041886	0.075623	0.049239	0.009833	0.070258	0.010025	0.059357
RELIANCE INFRA	0.048035	0.065117	-0.03888	0.048425	0.094656	0.063512	0.014823	0.085793	0.012158	0.073486
ITC	-0.04569	-0.03888	0.123452	-0.05723	-0.08467	-0.05267	0.00145	-0.09586	-0.00189	-0.07751
M&M	0.041886	0.048425	-0.05723	0.049375	0.08237	0.050941	0.010664	0.074027	0.015294	0.061941
GRASIM	0.075623	0.094656	-0.08467	0.08237	0.148523	0.096124	0.020567	0.135464	0.021991	0.114614
SBI	0.049239	0.063512	-0.05267	0.050941	0.096124	0.064158	0.012723	0.090454	0.010401	0.076816

WIPRO	0.009833	0.014823	0.00145	0.010664	0.020567	0.012723	0.005646	0.013808	0.007017	0.01222
TATA STEEL	0.07026	0.08579	-0.0959	0.07403	0.13546	0.09045	0.01381	0.13398	0.01017	0.11278
ONGC	0.01002	0.01216	-0.0019	0.01529	0.02199	0.01040	0.00702	0.01017	0.0138	0.00871
TATA MOTORS	0.05935	0.07348	-0.0775	0.06194	0.11461	0.07681	0.01222	0.11278	0.00871	0.09513

Table 3. Upper covariance matrix

STOCK	TATA POWER	RELIANCE INFRA	ITC	M&M	GRASIM
TATA POWER	0.21293	0.655396	-0.05235	-0.32236	-0.2534
RELIANCE INFRA	0.65539	2.078645	-0.16037	-0.86816	-0.70558
ITC	-0.0523	-0.16037	0.016754	0.145528	0.08063
M&M	-0.3223	-0.86816	0.145528	1.82057	0.824898
GRASIM	-0.253	-0.70558	0.08063	0.824898	0.469942
SBI	0.10740	0.326497	-0.0183	-0.03459	-0.09612
WIPRO	-0.1904	-0.50519	0.108887	1.472226	0.598989
TATA STEEL	-0.1329	-0.33455	0.087728	1.256278	0.491615
ONGC	-0.0312	-0.04828	0.019143	0.325719	0.144055
TATA MOTORS	-0.4914	-1.39089	0.223644	2.682954	1.187544

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SBI	0.107403	0.326497	-0.0183	-0.03459	-0.09612	0.071607	0.027155	0.041984	0.003937	-0.04269
WIPRO	-0.19038	-0.50519	0.108887	1.472226	0.598989	0.027155	1.239608	1.070363	0.262355	2.196997
TATA STEEL	-0.13287	-0.33455	0.087728	1.256278	0.491615	0.041984	1.070363	0.929748	0.229462	1.870106
ONGC	-0.0312	-0.0483	0.0191	0.3257	0.1441	0.0040	0.2624	0.2295	0.0724	0.4513
TATA MOTORS	-0.4914	-1.39089	0.223644	2.682954	1.187544	-0.04269	2.196997	1.870106	0.451286	4.025142

Let $k_i = 0.001$, $M = 0.8$, $m = 0.005, k = 8$, $x_i^0 = 0, \forall i = 1,2,3,............,10$.

Also let $\alpha = -0.8$; $\beta = 0.9$; $\gamma = 0.5$; $\lambda_1 = 0.4$; $\lambda_2 = 0.5$; $\lambda_3 = 0.3$.

Using these data, in this scenario let us solve the three portfolio optimization models $\langle 2 \rangle, \langle 3 \rangle, \langle 4 \rangle$. The solution is done by MATLAB and result obtained after 50 crossover and mutations in each model is shown in Table 4.

Table 4. Solutions & Constraints of models $\langle 2 \rangle, \langle 3 \rangle, \langle 4 \rangle$

Models	$\langle 2 \rangle$	$\langle 3 \rangle$	$\langle 4 \rangle$
Return	[-0.3820,1.1265]	[-0.3883,1.0744]	[-0.3749,1.1028]
Risk	[0.0524,0.3078]	[0.0379,0.3953]	[0.0416,0.2031]
Dividend	[0.8214,1.9853]	[1.0880,2.1498]	[1.1245,2.1178]
Portfolio			
TATA POWER	0.1259	0	0.1273
RELIANCE INFRA	0.1258	0.1129	0.1248
ITC	0	0.1298	0.1181
M&M	0	0.1418	0
GRASIM	0.1236	0.1129	0.1278
SBI	0.1223	0.1259	0.1245
WIPRO	0.1263	0	0
TATA STEEL	0.1272	0.1142	0.1276
ONGC	0.1281	0.1259	0.1173
TATA MOTORS	0.1208	0.1366	0.1326

In figure 1, the different portfolios for models 2 -4 are shown in the form of bar diagrams.
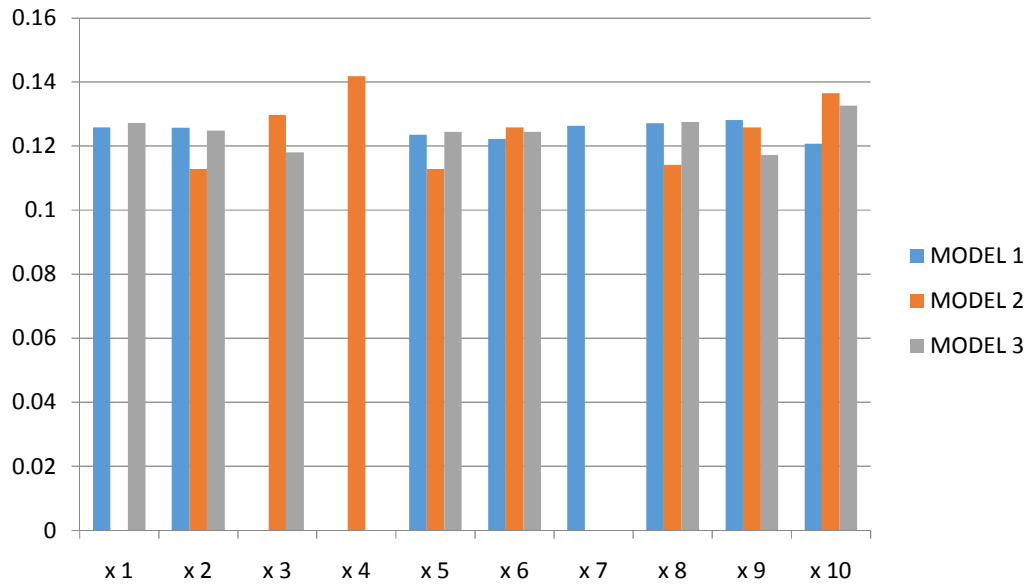
Figure 1. Allocation of capital for the three models

It is clear from the results shown in **Table 4** for models 2-4 that in model $\langle 2 \rangle$ the investor used the constraints pessimistically and optimistically in model $\langle 3 \rangle$, while in case of model $\langle 4 \rangle$, he was neither too optimistic nor too pessimistic, but a mixed one. This is prominent from the results of returns, risks and dividends obtained in each case:

RETURN: $[-0.3749, 1.1028] \subset [-0.3883, 1.1265]$,

RISK: $[0.0416, 0.2031] \subset [0.0379, 0.3953]$,

DIVIDEND: $[1.1245, 2.1178] \subset [0.8214, 2.1498]$.

From **Figure 1**, it is also evident that the investor is always allowed to handle with only 8 assets, and also each allocation is inside the desired margins.

6.   CONCLUSION

This article has introduced a new framework for fuzzy portfolio selection using interval valued fuzzy numbers. A new objective function has been defined as the ratio of the entropy of the allocated proportions of investor's wealth to the total transaction costs. In addition to the objective function, eight different constraints including constraints on return, risk and dividends have been introduced in the models. Three different models representing pessimistic, optimistic and balanced preference of the investors have been proposed. The return, covariance and dividends of the stocks are represented by interval numbers. The models are tested on stock data from BSE. The solution is done by the software MATLAB.

In future, the methodology presented here can be extended to the portfolio selection problems in fuzzy, hybrid and general uncertain environments. In addition, genetic algorithm, hybrid intelligent algorithm, tabu search, simulated annealing, ant-colony optimization and particle swam optimization may be employed to solve the non-linear programming problem.

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