ALPHA STABLE RANDOM FIELDS AND ADDITIVE ERROR

Rachid Sabre

Laboratory Biogéosciences CNRS,
University of Burgundy/Agrosup Dijon, France

ABSTRACT

This work studies the estimation of spectral density for random field (two-dimensional signal) when the spectral measure have certain mixture and the process is observed with a constant error. The objective of this paper is to give an estimator of the constant error by using the Jackson polynomial kernel. We show that the rate of convergence depends of size of sample and the behaviours of the spectral density at origin. Indeed the estimator converges rapidly when the spectral density is null at origin. Few long memory signals are taken here as example.

KEYWORDS

Spectral density; Jackson kernel; Stable random fields.

1. INTRODUCTION

This work considers the class of symmetric alpha stable signals which are known as signals having infinite energy. These signals have been developed in recent decades by several authors, including [1]-[13], to name a few.

The Gaussian density distribution remains a particular cases of alpha-stable distribution (α =2).

Alpha stable distribution is a better model for signals that are impulsive in nature. It is adapted for signals that their variance is large and the Gaussian can not usedfor modelling this process. Signals in this class contain high-pitched bursts or occasional spikes.

Symmetric alpha stable signals are used for modeling many phenomenons in several fields: : physics, biology, electronic and electrical engineering, hydrology, economies, communications and radar applications and signal image processing,…see [14]-[25].

In this work, we consider a symmetric α stable random field $Z = \{Z_{(n_1,n_2)}: (n_1, n_2) \in Z^2\}$ having the following integral representation:

$$Z_{(n_1,n_2)} = \int_{-\pi}^{\pi}\int_{-\pi}^{\pi} \exp[i(n_1\lambda_1 + n_2\lambda_2)]d\xi(\lambda_1, \lambda_2)$$

where $1 < \alpha < 2$ and ξ is a complex valued symmetric α-stable random measure on R^2 with independent and isotropic increments. The measure defined by $m(AxB) = |\xi(AxB)|_\alpha^\alpha$ (see [4]) is called "control" measure or spectral measure. The case where this measure is absolutely continuous with respect to Lebegue measure: $dm(x_1, x_2) = \phi(x_1, x_2)dx_1dx_2$ is considered in [4], [26], [27]. And the function ϕcalled the spectral density of \$Z\$ was already estimated by [4],

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when the time of the process is continuous, by [26] when the time of the process is discrete and by [27] when the time of the process is p-adic.

This paper considersa case, often encountred in pratice namely when we observe this random field withan unknown constant error:$X_{(n_1,n_2)} = a(n_1,n_2) + Z_{(n_1,n_2)}$Thus, the signal observed is $X_{(n_1,n_2)}$instead of the signal $Z_{(n_1,n_2)}$alone.We also consider a more general case: when the spectral measure is the sum of an absolutely continuous measure with respect to Lebesgue measure, a discrete measure and a finite umber of ebsolutely continous measure on several lines:

$$dm = \phi(x_1,x_2)dx_1dx_2 + \sum_{i=1}^{q} c_i\delta_{(w_{(1,i)},w_{(2,i)})} + \sum_{i=1}^{q} \varphi_k\delta_{(u_1,\ a_\mathrm{k}u_1+b_\mathrm{k})}$$

where δ is a Dirac measure, ϕ and $\varphi_k$are non-negative integrable and bounded functions. c_i is unknown positive real number and $w_{(1,i)}$,$w_{(2,i)}$, a_k and $b_\mathrm{k}$are unknown real numbers. Assume that $w_{(1,i)} \neq 0$ and $w_{(2,i)} \neq 0$. The estimation of the constant error when the process have one dimension is given in [28]. This mixed measure is encountered when, for example, the resistance of the soil is measured on agricultural land which has a continuous random measurement and when pebbles are randomly encountered the measurement reaches jumps which represents the discrete measure.When the measures are made in places where the passage of tractors is frequent. The rate of convergence will be studied particularly for spectral densities which are zero at the origin as$\phi(\lambda_1,\lambda_2) = \sin^{2k\alpha}\left(\frac{\lambda_1}{2}\right)\sin^{2k\alpha}\left(\frac{\lambda_2}{2}\right)g(\lambda_1,\lambda_2)$ and $\phi(\lambda_1,\lambda_2) = |\lambda_1\lambda_2|^{\beta}g(\lambda_1,\lambda_2)$. We show that the convergence speed is much faster depending on the value of the parameter of β.

This paper is organized as follows: The second section gives some proprties of Jackson polynomial kernel and an estimator of the constant a. We show that the estimator converges to $a$in probability and converges in $L_p(p < \alpha)$. Since, we assume that the spectral density of Z is vanishing at origin,we show that theestimator converges more rapidly to a in accordance with the values of β. The third sectionconsists in illustrating the results found through a numerical data.The last section, contains the concluding remarks, the potential applications and the open research problems.

2. ESTIMATION OF THE CONSTANT

This paper considers a (SαS) process where its spectral representation is :

$$Z_{(n_1,n_2)} = \int_{-\pi}^{\pi}\int_{-\pi}^{\pi} e^{i(n_1\lambda_1+n_2\lambda_2)}d\xi(\lambda_1,\lambda_2),$$

where ξ is a isotropic symmetric α-stable with independent increments.The measure defined by: $m(]s_1,t_1]]s_2,t_2]) = |\xi(s_1,t_1,) - \xi(s_2,t_2)|_\alpha^\alpha$ is Lebesgue-Stiel measure called *the spectral measure* (see [1], [4]) When m is absolutely continuous $d\mu = \phi(x_1,x_2)dx_1dx_2$, the function ϕ is called *the spectral density* of the process Z.

Let $Z_{(n1,n2)}$ observations of the process Z: $\left(Z_{(n_1,n_2)}\right)$ with $0 \leq n_1 \leq N_1 - 1$ and $0 \leq n_2 \leq N_2 - 1$, where N_1, N_2 satisfy: $N_1 - 1 = 2k(n_1 - 1)$ and $N_2 - 1 = 2k(n_2 - 1)$ with $n_1, n_2 \in N \quad k \in N \cup \left\{\frac{1}{2}\right\}$if $k = \frac{1}{2}$then$n_1 = 2n'_1 - 1,\ n'_1 \in N$ and $n_2 = 2n'_2 - 1, n'_2 \in N$.

The Jackson polynomial kernel is defined in [29], [11] and [26], as follows:

$$|H_N(\lambda_1,\lambda_2)|^{\alpha} = \left|A_{(N_1,N2)}H^{(N_1,N_2)}(\lambda_1,\lambda_2)\right|^{\alpha}$$

where $H^{(N_1,N_2)}(\lambda_1,\lambda_2) = \frac{1}{q_{k,n_1,n_2}}\left(\frac{\sin(\frac{n_1\lambda_1}{2})}{\sin(\frac{\lambda_1}{2})}\right)^{2k}\left(\frac{\sin(\frac{n_2\lambda_2}{2})}{\sin(\frac{\lambda_2}{2})}\right)^{2k}$

with$q_{k,n_1,n_2} = \left(\frac{1}{2\pi}\right)^2\int_{-\pi}^{\pi}\int_{-\pi}^{\pi}\left(\frac{\sin(\frac{n_1\lambda_1}{2})}{\sin(\frac{\lambda_1}{2})}\right)^{2k}\left(\frac{\sin(\frac{n_2\lambda_2}{2})}{\sin(\frac{\lambda_2}{2})}\right)^{2k} d\lambda_1 d\lambda_2$

In addition, we have $A_{(N_1,N_2)} = (B_{\alpha,N_1,N_2})^{\frac{-1}{\alpha}}$ with $B_{\alpha,N_1,N_2} = \int_{-\pi}^{\pi}\int_{-\pi}^{\pi}\left|H^{(N_1,N_2)}(\lambda_1,\lambda_2)\right|^{\alpha} d\lambda_1 d\lambda_2$.

The estimator:

In this paper, we constructthe following estimate of the error a:

$$\hat{a} = \frac{A_{(N_1,N2)}}{H_N(0,0)}\sum_{n'=-k(n_1-1)}^{k((n_1-1)}\sum_{n''=-k(n_2-1)}^{k((n_2-1)} X(n'+k(n_1-1,n''+k(n_2-1)).(1)$$

We start by showing that the estimator $\hat{a}$converges to a in probability and converges in $L_p(p < \alpha)$. Then we show that theses convergences are faster for signals whose spectral density vanishes at the origin. This means that the etimator is slowed down by the disturbance of the energy at the origin.To lighten the formulas and make the paper easy to read, we delete in the rest of this paper the last term of the expression of the spectral measure.Thus, henceforth the considered spectral measure becomes:

$$dm = \phi(x_1,x_2)dx_1dx_2 + \sum_{i=1}^{q} c_i\delta_{(w_{(1,i)},w_{(2,i)})}$$

Citing nowtwo lemmas given propertiesof Jackson polynomial kernel that we will use later. Their proofs are given [26], [29].

Lemma 1

The function$H^{(N_1,N_2)}(\lambda_1,\lambda_2)$ can be written as follows:

$$H^{(N_1,N_2)}(\lambda_1,\lambda_2) = \sum_{m_1=-k(n_1-1)}^{k(n_1-1)}\sum_{m_2=-k(n_2-1)}^{k(n_2-1)} h_k\left(\frac{m_1}{n_1}\right)cos(m_1\lambda_1)h_k\left(\frac{m_2}{n_2}\right)cos(m_1\lambda_1),$$

where h_k is a even positive function.

Lemma 2.

Let

$$B'_{\alpha,N_1,N_2} = \int_{-\pi}^{\pi}\int_{-\pi}^{\pi}\left|\frac{sin\frac{n_1\lambda_1}{2}}{sin\frac{\lambda_1}{2}}\right|^{2k\alpha}\left|\frac{sin\frac{n_2\lambda_2}{2}}{sin\frac{\lambda_2}{2}}\right|^{2k\alpha} d\lambda_1 d\lambda_2$$

and

$$J_{N_1,N_2,\alpha} = \int_{-\pi}^{\pi}\int_{-\pi}^{\pi} |(u_1,u_2)|^\gamma |H_{N_1,N_2}(\lambda_1,\lambda_2)|^\alpha d\lambda_1 d\lambda_2,$$

where $\gamma \in]0,2]$. *Then we have the following inqualities,*

$$B'_{\alpha,N_1,N_2} \begin{cases} \geq \left(2\pi\left(\frac{2}{\pi}\right)^{2k\alpha}\right)^2 n_1^{2k\alpha-1} n_2^{2k\alpha-1} & if \;\; 0 < \alpha < 2 \\ \leq \left(\frac{4\pi k\alpha}{2k\alpha-1}\right)^2 n_1^{2k\alpha-1} n_2^{2k\alpha-1} & if \;\; \frac{1}{2k} < \alpha < 2 \end{cases}$$

and

$$J_{N_1,N_2,\alpha} \leq \left(\frac{\pi^{\gamma+2k\alpha}}{2^{2k\alpha}(\gamma-2k\alpha)}\right)^2 \frac{1}{n_1^{2k\alpha-1}}\frac{1}{n_2^{2k\alpha-1}} \quad if \;\; \frac{1}{2k} < \alpha < \frac{\gamma+1}{2k},$$

$$J_{N_1,N_2,\alpha} \leq \left(\frac{2k\alpha\pi^{\gamma+2k\alpha}}{2^{2k\alpha}(\gamma+1)(2k\alpha-\gamma-1)}\right)^2 \frac{1}{(n_1 n_2)^\gamma} \;\; if \;\; \frac{\gamma+1}{2k} < \alpha < 2.$$

Theorem 1

Let p a real number such that $0 < p < \alpha$. *We have*

$$|\hat{a} - a|^p = O\left(\frac{1}{(n_1 n_2)^{\frac{p}{\alpha}}}\right)$$

Proof Using the spectral representation of the process, we obtain

$$\hat{a} = \frac{A_{(N_1,N_2)}}{H_{(N_1,N2)}(0,0)} \sum_{n'=-k(n_1-1)}^{k((n_1-1)} \sum_{n''=-k(n_2-1)}^{k((n_2-1)} h_k\left(\frac{n'}{n_1},\frac{n''}{n_2}\right) \int_{-\pi}^{\pi}\int_{-\pi}^{\pi} exp[i([n'+k(n_1-1)]\lambda_1)]$$
$$exp[i([n''+k(n_2-1)]\lambda_2)] d\xi(\lambda_1,\lambda_2) + a$$

From [1], we can writte the characteristic function of $(\hat{a} - a)$:

$E\exp[i\Re e\bar{r}(\hat{a} - a)] =$

$$exp - C_\alpha |r|^\alpha \int_{-\pi}^{\pi} \left| \frac{A_{(N_1,N_2)}}{H_{(N_1,N_2)}(0,0)} \sum_{n\prime=-k(n_1-1)}^{k((n_1-1)} \sum_{n\prime\prime=-k(n_2-1)}^{k((n_2-1)} h_k\left(\frac{n'}{n_1},\frac{n''}{n_2}\right) e^{in\prime\lambda_1} e^{in\prime\prime\lambda_2} \right|^\alpha d\xi(\lambda_1,\lambda_2)$$

where $r = r_1 + ir_2$. It is easy to show that:

$E\exp[i\Re e\bar{r}(\hat{a} - a)] = \exp\left(-C_\alpha |r|^\alpha \psi_{(N_1,N_2)}\right)$, where $\psi_{(N_1,N_2)} = \psi_{N_1,N_2,1} + \psi_{N_1,N_2,2}$ with
$\psi_{N_1,N_2,1} = \int_{-\pi}^{\pi}\int_{-\pi}^{\pi} \frac{\left|H_{(N_1,N_2)}(\lambda_1,\lambda_2)\right|^\alpha}{\left|H_{(N_1,N_2)}(0,0)\right|^\alpha} \phi(\lambda_1,\lambda_2) d\lambda_1 d\lambda_2$ *and*

$$\psi_{N_1,N_2,2} = \sum_{i=1}^{q} c_i \frac{\left|H_{(N_1,N_2)}(w_{i,1}, w_{i,2})\right|^{\alpha}}{\left|H_{(N_1,N_2)}(0,0)\right|^{\alpha}}.$$

The function ϕ being bounded on $[-\pi,\pi]^2$ and $|H_{(N_1,N_2)}(.,.)|^{\alpha}$ being a kernel, it can be shown that $\int_{-\pi}^{\pi}\int_{-\pi}^{\pi} \left|H_{(N_1,N_2)}(\lambda_1,\lambda_2)\right|^{\alpha}\phi(\lambda_1,\lambda_2)d\lambda_1 d\lambda_2$ is converging to $\phi(0,0)$. On the other hand, from lemma 2, we have:

$$\frac{1}{|H_{(N_1,N_2)}(0,0)|^{\alpha}} = \frac{B'_{\alpha,N_1,N_2}}{n_1^{2k\alpha}n_2^{2k\alpha}} = O\left(\frac{1}{n_1 n_2}\right) \qquad (2)$$

Therefore $\psi_{N_1,N_2,1}$ converges to 0.

$$\psi_{N_1,N_2,2} \le \sum_{i=1}^{q} \frac{c_i}{B'_{\alpha,N_1,N_2}} \frac{1}{\left|\sin\left[\frac{w_{i,1}}{2}\right]\sin\left[\frac{w_{i,2}}{2}\right]\right|^{2k\alpha}} \frac{B'_{\alpha,N_1,N_2}}{n_1^{2k\alpha}n_2^{2k\alpha}}.$$

Therefore $\psi_{N_1,N_2,2} = O\left(\frac{1}{n_1^{2k\alpha}n_2^{2k\alpha}}\right)$. Thus

$$\psi_{(N_1,N_2)} = O\left(\frac{1}{n_1 n_2}\right).$$

Consequently, the characteristic function of $\hat{a}-a$ converges to 1 when N_1 and N_2 tend to infinity. We deduce that$\hat{a}$ convergences to $a$in probability.

In order to study the convergence in L_p where $0 < p < \alpha$, we put

$$D_p = \Re e \int_{-\infty}^{\infty}\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{1-e^{ir\cos\theta}}{|r|^{1+p}} dr d\theta.$$

Assuming now $r = \varepsilon r'$, $\quad \theta = \tau' - \tau_0$

$$D_p = \Re e \int_{-\infty}^{\infty}\int_{-\frac{\pi}{4}+\tau_0}^{\frac{\pi}{4}+\tau_0} \frac{1-e^{i\varepsilon r'\cos(\tau'-\tau_0)}}{|\varepsilon|^{1+p}|r'|^{1+p}} \varepsilon dr'\tau'$$

$$D_p|x|^p = \Re e \int_{0}^{\infty}\int_{-\frac{\pi}{4}+\tau_0}^{\frac{\pi}{4}+\tau_0} \frac{1-e^{i\Re e(\bar{t}x)}|}{t|^{1+p}} d|t|d\theta' - \Re e \int_{-\infty}^{0}\int_{-\frac{\pi}{4}+\tau_0}^{\frac{\pi}{4}+\tau_0} \frac{1-e^{i\Re e(\bar{t}x)}}{|t|^{1+p}} d|t|d\theta'.$$

Let us substitute x by $\hat{a}-a$, we have

$$D_p E|\hat{a}-a|^p = \int_{-\infty}^{\infty}\int_{-\frac{\pi}{4}+\tau_0}^{\frac{\pi}{4}+\tau_0} \frac{1-e^{-C_\alpha|t|^{\alpha}\psi_{(N_1,N_2)}}}{|t|^{1+p}} d|t|d\theta' = \frac{\pi}{2}\int_{-\infty}^{\infty} \frac{1-e^{-C_\alpha|t|^{\alpha}\psi_N}}{|t|^{1+p}} dt.$$

Let $u = t[\psi_{(N_1,N_2)}]^{1\alpha}$ and using (2), we obtain

$$\frac{2}{\pi} C_{p,\alpha} E|\hat{a}-a|^p = \left(\psi_{(N_1,N_2)}\right)^{\frac{p}{\alpha}} = O\left(\frac{1}{n_1^{p/\alpha} n_2^{p/\alpha}}\right). \qquad (3)$$

where $C_{p,\alpha} = R_p F_{p,\alpha}^{-1}(C_\alpha)^{-\frac{p}{\alpha}}$

with $R_p = \int \frac{1-\cos(u)}{|u|^{1+p}} du$ *and* $F_{p,\alpha} = \int \frac{1-e^{-|u|^\alpha}}{|u|^{\frac{1+p}{\alpha}}} du.$

Improvement of the rate of convergence

We take the case where the spectral density vanishes at the origin. The following theorems will show that the speed of convergence is better.

Theorem 2

Assume that the spectral density is satisfying:

$$\phi(\lambda_1,\lambda_2) = |\lambda_1|^\beta |\lambda_2|^\beta g(\lambda_{,1},\lambda_2)$$

where $\beta \in]0,2k\alpha-1[, \lambda_1,\lambda_2 \in [-\pi,\pi]$ *and* $g(\lambda_1,\lambda_2)$ *is a bounded function on* $[-\pi,\pi]^2$, *continuous in neighborhood of* $(0,0)$ *and* $g(0,0) \neq 0$. *Then*

$$2^{8kp} L \leq \lim_{N_1,N_2\to\infty} (n_1 n_2)^{\frac{p(\beta+1)}{\alpha}} E|\hat{a}-a|^p \leq \pi^{8kp} L,$$

where L is the following constant:

$$L = \frac{\pi}{2C_{p,\alpha}} \left[g(0,0) \int_{-\infty}^{\infty} \frac{\left|\sin\frac{v}{2}\right|^{2k\alpha}}{|v|^{2k\alpha-\beta}} dv \right]^{\frac{2p}{\alpha}}.$$

Proof:

From (2), the function $\psi_{(N_1,N_2)}$ can be written as:

$$\psi_{(N_1,N_2)} = (n_1 n_2)^{-2k\alpha} \int_{-\pi}^{\pi}\int_{-\pi}^{\pi} \left|\frac{\sin\frac{n_1\lambda_1}{2}}{\sin\frac{\lambda_1}{2}}\right|^{2k\alpha} \left|\frac{\sin\frac{n_2\lambda_2}{2}}{\sin\frac{\lambda_2}{2}}\right|^{2k\alpha} |\lambda_1\lambda_2|^\beta g(\lambda_1,\lambda_2) d\lambda_1 d\lambda_2$$
$$+(n_1 n_2)^{-2k\alpha} \sum_{i=1}^{q} c_i \left| \frac{\sin\left[\frac{n_1 w_{i,1}}{2}\right]}{\sin\left[\frac{w_{i,1}}{2}\right]} \frac{\sin\left[\frac{n_2 w_{i,2}}{2}\right]}{\sin\left[\frac{w_{i,2}}{2}\right]} \right|^{2k\alpha}$$

Using the following inequality:

$$|\sin x2| \geq x\pi \qquad 0 \leq x \leq \pi, \tag{4}$$

we maximize $\psi_{(N_1,N_2)}$ as follows:

$$\psi_{(N_1,N_2)} \leq \left(\pi^{4k\alpha}\right)^2 (n_1 n_2)^{-2k\alpha} \int_{-\pi}^{\pi}\int_{-\pi}^{\pi} \frac{\left|\sin\frac{n_1\lambda_1}{2}\right|^{2k\alpha}}{|\lambda_1|^{2k\alpha-\beta}} \frac{\left|\sin\frac{n_2\lambda_2}{2}\right|^{2k\alpha}}{|\lambda_2|^{2k\alpha-\beta}} g(\lambda_1,\lambda_2) d\lambda_1 d\lambda_2$$

$$+(n_1n_2)^{-2k\alpha}\sum_{i=1}^{q} c_i\left|\frac{1}{\sin\left[\frac{w_{i,1}}{2}\right]}\frac{1}{\sin\left[\frac{w_{i,2}}{2}\right]}\right|^{2k\alpha}.$$

Putting $n_1\lambda_1 = u_1$ and $n_2\lambda_2 = u_2$, we have

$$\psi_{(N_1,N_2)} \leq \left(\pi^{4k\alpha}\right)^2(n_1n_2)^{-1-\beta}\int_{-\infty}^{\infty}\frac{\left|\sin\frac{u_1}{2}\right|^{2k\alpha}}{|u_1|^{2k\alpha-\beta}}\frac{\left|\sin\frac{u_2}{2}\right|^{2k\alpha}}{|u_2|^{2k\alpha-\beta}}\,g\left(\frac{u_1}{n_1},\frac{u_2}{n_2}\right)du_1du_2$$
$$+\frac{(n_1n_2)^{-2k\alpha+1+\beta}}{(\pi^{4k\alpha})^2}\sum_{i=1}^{q} c_i\left|\frac{1}{\sin\left[\frac{w_{i,1}}{2}\right]}\frac{1}{\sin\left[\frac{w_{i,2}}{2}\right]}\right|^{2k\alpha}.$$

FromLemma 2, andthe theorem of Lebesgue's dominated convergence we obtain that:

$$\lim_{N_1,N_2\to\infty}(n_1n_2)^{p(\beta+1)\alpha}\left(\psi_{(N_1,N_2)}\right)^{p\alpha} \leq \left(\pi^{4kp}\left(g(0,0)\int_{-\infty}^{+\infty}|\sin u2|^{2k\alpha}|u|^{2k\alpha-\beta}du\right)^{p\alpha}\right)^2.$$

Thus $\psi_{(N_1,N_2)}$ converges to zero.Using the following inequality

$$|\sin x| \leq |x| \qquad \forall x \in [-\pi,\pi], \tag{5}$$

we obtain:

$$\psi_{(N_1,N_2)} \geq 2^{8k\alpha}(n_1n_2)^{-2k\alpha}\int_{-\pi}^{\pi}\int_{-\pi}^{\pi}\frac{\left|\sin\frac{n_1\lambda_1}{2}\right|^{2k\alpha}}{|\lambda_1|^{2k\alpha-\beta}}\frac{\left|\sin\frac{n11\lambda_2}{2}\right|^{2k\alpha}}{|\lambda_2|^{2k\alpha-\beta}}\,g(\lambda_1,\lambda_2)d\lambda_1d\lambda_2$$
$$+(n_1n_2)^{-2k\alpha}\sum_{i=1}^{q} c_i\left|\frac{\sin\left[\frac{n_1w_{i,1}}{2}\right]}{\sin\left[\frac{w_{i,1}}{2}\right]}\frac{\sin\left[\frac{n_2w_{i,2}}{2}\right]}{\sin\left[\frac{w_{i21}}{2}\right]}\right|^{2k\alpha}.$$

$$\psi_{(N_1,N_2)} \geq 2^{8k\alpha}(n_1n_2)^{-\beta-1}\left[\int_{-\pi}^{\pi}\int_{-\pi}^{\pi}\frac{\left|\sin\frac{u_1}{2}\right|^{2k\alpha}}{|u_1|^{2k\alpha-\beta}}\frac{\left|\sin\frac{u_2}{2}\right|^{2k\alpha}}{|u_1|^{2k\alpha-\beta}}\,g\left(\frac{u_1}{n_1},\frac{u_2}{n_2}\right)du_1du_2 + R_n\right],$$

where $R_n = \frac{(n_1n_2)^{-2k\alpha+\beta+1}}{2^{8k\alpha}}\sum_{i=1}^{q} c_i\left|\frac{\sin\left[\frac{n_1w_{i,1}}{2}\right]}{\sin\left[\frac{w_{i,1}}{2}\right]}\frac{\sin\left[\frac{n_2w_{i,2}}{2}\right]}{\sin\left[\frac{w_{i,2}}{2}\right]}\right|^{2k\alpha}.$

Since R_n converges to zero, the equality (5) gives

$$\lim_{N_1,N_2\to\infty}(n_1n_2)^{\frac{p(\beta+1)}{\alpha}}\left(\psi_{(N_1,N_2)}\right)^{\frac{p}{\alpha}} \geq 2^{8kp}\left(g(0,0)\int_{-\infty}^{+\infty}\left|\sin(\tfrac{u}{2})\right|^{2k\alpha}/|u|^{2k\alpha-\beta}du\right)^{2p/\alpha}.$$

We arrive at the result of this theorem and this because of equality (3).

Theorem 3

Assuming that the spectral density satisfies :

$\phi(\lambda_1,\lambda_2) = \left(sin\frac{n_2\lambda_2}{2}sin\frac{n_2\lambda_2}{2}\right)^{2k\alpha}g(\lambda_1,\lambda_2)$*where* g *is real function, integrable on* $[-\pi,\pi]$ *and*

$g(0,0) \neq 0. \, Then$

$$cte\left(\frac{\pi}{2C_{p,\alpha,}}\right)^2 \int_{-\pi}^{\pi}\int_{-\pi}^{\pi} g(\lambda_1,\lambda_2)d\lambda_1 d\lambda_2 \leq \lim_{N_1,N_2\to\infty}(n_1n_2)^{2kp}E|\hat{a}-a|^p \leq$$

$$\left(\frac{\pi}{2C_{p,\alpha,}}\right)^2 \int_{-\pi}^{\pi}\int_{-\pi}^{\pi} g(\lambda_1,\lambda_2)d\lambda_1 d\lambda_2 + \sum_{i=1}^{q} c_i \left|\frac{sin\left[\frac{n_1w_{i,1}}{2}\right]}{sin\left[\frac{w_{i,1}}{2}\right]}\frac{sin\left[\frac{n_2w_{i,2}}{2}\right]}{sin\left[\frac{w_{i,2}}{2}\right]}\right|^{2k\alpha}$$

Proof: $From the definition of \, \psi_{(N_1,N_2)}$ and(2), we have

$$\psi_{(N_1,N_2)} = (n_1n_2)^{-2k\alpha}\int_{-\pi}^{\pi}\int_{-\pi}^{\pi}\left|\sin\frac{n_1\lambda_1}{2}\sin\frac{n_1\lambda_1}{2}\right|^{2k\alpha} g(\lambda_1,\lambda_2)d\lambda_1 d\lambda_2$$

$$+(n_1n_2)^{-2k\alpha}\sum_{i=1}^{q} c_i \left|\frac{\sin\left[\frac{n_1w_{i,1}}{2}\right]}{\sin\left[\frac{w_{i,1}}{2}\right]}\frac{\sin\left[\frac{n_1w_{i,2}}{2}\right]}{\sin\left[\frac{w_{i,2}}{2}\right]}\right|^{2k\alpha}.$$

$$\psi_{(N_1,N_2)} \leq (n_1n_2)^{-2k\alpha}\int_{-\pi}^{\pi}\int_{-\pi}^{\pi} g(\lambda_1,\lambda_2)d\lambda_1 d\lambda_2$$

$$+(n_1n_2)^{-2k\alpha}\sum_{i=1}^{q} c_i \left|\frac{\sin\left[\frac{n_1w_{i,1}}{2}\right]}{\sin\left[\frac{w_{i,1}}{2}\right]}\frac{\sin\left[\frac{n_2w_{i,2}}{2}\right]}{\sin\left[\frac{w_{i,2}}{2}\right]}\right|^{2k\alpha}$$

The lemma 2:

$$\lim_{N_1,N_2\to\infty}\psi_{(N_1,N_2)}(n_1n_2)^{2k\alpha} \leq \int_{-\pi}^{\pi}\int_{-\pi}^{\pi} g(\lambda_1,\lambda_2)d\lambda_1 d\lambda_2 + \sum_{i=1}^{q} c_i \left|\frac{1}{\sin\left[\frac{w_{i,1}}{2}\right]}\frac{1}{\sin\left[\frac{w_{i,2}}{2}\right]}\right|^{2k\alpha}.$$

To minimize ψ_((N_1,N_2)) , using the proprities of sinus we obtain

$$\psi_{(N_1,N_2)} \geq (n_1n_2)^{-2k\alpha}\int_{-\pi}^{\pi}\int_{-\pi}^{\pi}\left[\left(\sin\frac{n_1\lambda_1}{2}\sin\frac{n_2\lambda_2}{2}\right)^2\right]^{[k\alpha]+1} g(\lambda_1,\lambda_2)d\lambda_1 d\lambda_2$$

$$+(n_1n_2)^{-2k\alpha}\sum_{i=1}^{q} c_i \left|\frac{\sin\left[\frac{n_1w_{i,1}}{2}\right]}{\sin\left[\frac{w_{i,1}}{2}\right]}\frac{\sin\left[\frac{n_1w_{i,2}}{2}\right]}{\sin\left[\frac{w_{i,2}}{2}\right]}\right|^{2k\alpha}$$

where $[k\alpha]$ represents the integer part of $k\alpha$, we obtain

$$\psi_{(N_1,N_2)} \geq (n_1n_2)^{-2k\alpha}\int_{-\pi}^{\pi}\int_{-\pi}^{\pi}\left[\cos\frac{n_1\lambda_1-n_2\lambda_2}{2}-\cos\frac{n_1\lambda_1+n_2\lambda_2}{2}\right]^{2[k\alpha]+2} g(\lambda_1,\lambda_2)d\lambda_1 d\lambda_2$$

$$+(n_1n_2)^{-2k\alpha}\sum_{i=1}^{q} c_i \left|\frac{\sin\left[\frac{n_1w_{i,1}}{2}\right]}{\sin\left[\frac{w_{i,1}}{2}\right]}\frac{\sin\left[\frac{n_1w_{i,2}}{2}\right]}{\sin\left[\frac{w_{i,2}}{2}\right]}\right|^{2k\alpha}$$

Using the binomial formula we have:

$$\left[\cos\frac{n_1\lambda_1-n_2\lambda_2}{2}-\cos\frac{n_1\lambda_1+n_2\lambda_2}{2}\right]^{2[k\alpha]+2} =$$

$$\sum_{r=1}^{2[k\alpha]+2} C_{2[k\alpha]+2}^{r}(-1)^r\left(\cos^r\frac{n_1\lambda_1-n_2\lambda_2}{2}\cos^{(2[k\alpha]+2-r)}\frac{n_1\lambda_1+n_2\lambda_2}{2}\right)$$

Using again the binomial formula, we obtain:

$$\cos^r(a) = \left(e^{a} + e^{-ia}2\right)^r = 12^r \sum_{j=0}^{r} C_r^j e^{ija} e^{-i(r-j)a}.$$

Hence

$$\left[\cos\frac{n_1\lambda_1 - n_2\lambda_2}{2} - \cos\frac{n_1\lambda_1 + n_2\lambda_2}{2}\right]^{2[k\alpha]+2} = \sum_{r=1}^{2[k\alpha]+2} C_{2[k\alpha]+2}^r (-1)^r \left(\frac{1}{2^r}\sum_{j=0}^{r} C_r^j e^{ij\frac{n_1\lambda_1 - n_2\lambda_2}{2}} e^{-i(r-j)\frac{n_1\lambda_1 + n_2\lambda_2}{2}}\right)$$

$$\left[\cos\frac{n_1\lambda_1 - n_2\lambda_2}{2} - \cos\frac{n_1\lambda_1 + n_2\lambda_2}{2}\right]^{2[k\alpha]+2} = \sum_{r=1}^{2[k\alpha]+2} C_{2[k\alpha]+2}^r (-1)^r e^{-i(r)\frac{n_2\lambda_2}{2}} \left(12^r \sum_{j=0}^{r} C_r^j e^{i(2jr)\frac{n_1\lambda_1}{2}}\right)$$

$$\int_{-\pi}^{\pi}\int_{-\pi}^{\pi}\left[\cos\frac{n_1\lambda_1 - n_2\lambda_2}{2} - \cos\frac{n_1\lambda_1 + n_2\lambda_2}{2}\right]^{2[k\alpha]+2} g(\lambda_1,\lambda_2)d\lambda_1 d\lambda_2 = \sum_{r=1}^{2[k\alpha]+2} C_{2[k\alpha]+2}^r (-1)^r \int_{-\pi}^{\pi} e^{-i(r)\frac{n_2\lambda_2}{2}} \left(\frac{1}{2^r}\sum_{j=0}^{r} C_r^j \int_{-\pi}^{\pi} e^{i(2jr)\frac{n_1\lambda_1}{2}} g(\lambda_1,\lambda_2)d\lambda_1\right) d\lambda_2$$

The function $g(.,\lambda_2)$ being even for all $\lambda_2 \in [-\pi,\pi]$, we have

$$\int_{-\pi}^{\pi}\int_{-\pi}^{\pi}\left[\cos\frac{n_1\lambda_1 - n_2\lambda_2}{2} - \cos\frac{n_1\lambda_1 + n_2\lambda_2}{2}\right]^{2[k\alpha]+2} g(\lambda_1,\lambda_2)d\lambda_1 d\lambda_2 = \sum_{r=1}^{2[k\alpha]+2} C_{2[k\alpha]+2}^r (-1)^r \int_{-\pi}^{\pi} e^{-i(r)\frac{n_2\lambda_2}{2}} \left(\frac{1}{2^r}\sum_{j=0}^{r} C_r^j \int_{-\pi}^{\pi} \cos\left((2jr)\frac{n_1\lambda_1}{2}\right) g(\lambda_1,\lambda_2)d\lambda_1\right) d\lambda_2$$

The function $g(\lambda_1,.)$ being even for all $\lambda_1 \in [-\pi,\pi]$, we have

$$\int_{-\pi}^{\pi}\int_{-\pi}^{\pi}\left[\cos\frac{n_1\lambda_1 - n_2\lambda_2}{2} - \cos\frac{n_1\lambda_1 + n_2\lambda_2}{2}\right]^{2[k\alpha]+2} g(\lambda_1,\lambda_2)d\lambda_1 d\lambda_2 = \sum_{r=1}^{2[k\alpha]+2} C_{2[k\alpha]+2}^r (-1)^r \int_{-\pi}^{\pi} \cos\left(-\mathrm{i}r\frac{n_2\lambda_2}{2}\right) \left(\frac{1}{2^r}\sum_{j=0}^{r} C_r^j \int_{-\pi}^{\pi} \cos\left((2jr)\frac{n_1\lambda_1}{2}\right) g(\lambda_1,\lambda_2)d\lambda_1\right) d\lambda_2$$

Since $\frac{1}{2^r}\sum_{j=0}^{r} C_r^j \int_{-\pi}^{\pi} \cos\left((2jr)\frac{n_1\lambda_1}{2}\right) g(\lambda_1,\lambda_2)d\lambda_1 d\lambda_2$ converges to $\int_{-\pi}^{\pi} g(\lambda_1,\lambda_2)d\lambda_1 d\lambda_2$ uniformly in λ_2 when r is even and converges to 0 when r is odd. Therefore $\int_{-\pi}^{\pi}\int_{-\pi}^{\pi}\left[\cos\frac{n_1\lambda_1 - n_2\lambda_2}{2} - \cos\frac{n_1\lambda_1 + n_2\lambda_2}{2}\right]^{2[k\alpha]+2} g(\lambda_1,\lambda_2)d\lambda_1 d\lambda_2$

Converges to $\sum_{p=1}^{\frac{2[k\alpha]+2}{2}} C_{2[k\alpha]+2}^{2p} 12^{2p} \int_{-\pi}^{\pi}\int_{-\pi}^{\pi} g(\lambda_1,\lambda_2)d\lambda_1 d\lambda_2$.

Since $\lim_{N_1,N_2\to\infty}(n_1 n_2)^{-2k\alpha}\sum_{i=1}^{q} c_i \left|\frac{\sin\left[\frac{n_1 w_{i,1}}{2}\right]}{\sin\left[\frac{w_{i,1}}{2}\right]}\frac{\sin\left[\frac{n_2 w_{i,2}}{2}\right]}{\sin\left[\frac{w_{i,2}}{2}\right]}\right|^{2k\alpha} = 0$. The similar arguments are used for showing that

$$\lim_{N_1,N_2\to\infty}\left(\psi_{(N_1,N_2)}\right)^{\frac{p}{\alpha}}(n_1 n_2)^{2kp} \geq cte\left(\int_{-\pi}^{\pi} g(\lambda_1,\lambda_2)d\lambda_1 d\lambda_2\right)^{p\alpha}. \quad (6)$$

Theorem 4

Assume that spectral density satisfies:

$$\phi(\lambda_1\lambda_2) = |\lambda_1|^{\beta}|\lambda_2|^{\beta}g(\lambda_1,\lambda_2)$$

where g is real positive function and$\beta > 2k\alpha - 1$. Suppose that $g$is bounded on $[-\pi,\pi]^2$. Then

$$cte \times R \leq \lim_{N_1,N_2\to\infty}(n_1n_2)^{2kp}E|\hat{a}-a|^p \leq R$$

where $R = \left(\frac{\pi}{2C_{p,\alpha}}\right)^2 \left(\int_{-\pi}^{\pi}\int_{-\pi}^{\pi}\left(\frac{|\lambda_1|^{\beta}}{\left|sin\frac{\lambda_1}{2}\right|^{2k\alpha}}\frac{|\lambda_2|^{\beta}}{\left|sin\frac{\lambda_2}{2}\right|^{2k\alpha}}g(\lambda_1,\lambda_2)d\lambda_1d\lambda_2\right)^{p\alpha}\right.$.

Proof:

Define the continuous function l as follows:

$$l(\lambda) = \begin{cases} \pi^{2k\alpha} if|\lambda| > \pi \\ |\lambda|^{2k\alpha}|sin\frac{\lambda}{2}|^{2k} & if\ 0 < |\lambda| \leq \pi \\ 2^{2k\alpha} & if \quad \lambda = 0. \end{cases}$$

The function $\psi_{(N_1,N_2)}$ can be written as:

$$\psi_{(N_1,N_2)} = (n_1n_2)^{-2k\alpha}\int_{-\pi}^{\pi}\int_{-\pi}^{\pi}\left|\frac{\sin\frac{n_1\lambda_1}{2}}{\sin\frac{\lambda_1}{2}}\right|^{2k\alpha}\sin^{2k\alpha}\left(\frac{\lambda_1}{2}\right)\left|\frac{\sin\frac{n_2\lambda_2}{2}}{\sin\frac{\lambda_2}{2}}\right|^{2k\alpha}\sin^{2k\alpha}\left(\frac{\lambda_2}{2}\right)h(\lambda_1,\lambda_2)d\lambda_1d\lambda_2 + (n_1n_2)^{-2k\alpha}\sum_{i=1}^{q}c_i\left|\frac{\sin\left[\frac{n_1w_{i,1}}{2}\right]}{\sin\left[\frac{w_{i,1}}{2}\right]}\frac{\sin\left[\frac{n_1w_{i,2}}{2}\right]}{\sin\left[\frac{w_{i,2}}{2}\right]}\right|^{2k\alpha}$$

where $h(\lambda_1,\lambda_2) = l(\lambda_1)l(\lambda_2)|\lambda_1\lambda_2|^{\beta-2k\alpha}g(\lambda_1,\lambda_2)$. The function h is integrable on $[-\pi,\pi]^2$. Thus

$$\int_{-\pi}^{\pi}\int_{-\pi}^{\pi}h(\lambda_1,\lambda_2)d\lambda_{1_2}d\lambda \leq \sup(g)\int_{-\pi}^{\pi}\int_{-\pi}^{\pi}l(\lambda_1)l(\lambda_2)|\lambda_1\lambda_2|^{\beta-2k\alpha}d\lambda_1d\lambda_2.$$

Using the inequality (6), we obtain $l(\lambda_1) \leq \pi^{2k\alpha}$ and $l(\lambda_2) \leq \pi^{2k\alpha}$. Thus

$$\int_{-\pi}^{\pi}\int_{-\pi}^{\pi}h(\lambda_1,\lambda_2)d\lambda_1d\lambda_2 \leq 4\pi^{4k\alpha}\sup(g)\int_0^{\pi}\int_0^{\pi}(\lambda_1\lambda_2)^{\beta-2k\alpha}d\lambda_1d\lambda_2.$$

Since $\beta > 2k\alpha - 1$, the function h is integrable. From (3) and the theorem 1, the result is obtained.

3. NUMERICAL STUDIES

We give the simulation of the studied process:

$$Z_{n_1,n_2} = \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} e^{i(n_1\lambda_1+n_2\lambda_2)} d\xi(\lambda_1,\lambda_2), \qquad (7)$$

where $1 < \alpha < 2$ and ξ is a complex symmetric α-stable measure on R with independent and isotropic increments and with control measure m such that $dm = \phi(x_1,x_2)dx_1dx_2$.

For that, we use the series representations given by [30]). Therein the authors have shown that the process Z defined by (8) can be expressed as follows:

$$Z_{n_1,n_2} = C_\alpha(\int \phi(x_1,x_2)dx_1dx_2)^{1\alpha} \sum_{k=1}^{\infty} \varepsilon_k \Gamma_k^{-1\alpha} e^{i(n_1V_{1,k}+n_2V_{2,k})} e^{i\theta_k} \qquad (8)$$

where

- ε_k is a sequence of i.i.d. random variables such as $P[\varepsilon_k = 0] = P[\varepsilon_k = 1] = \frac{1}{2}$,
- Γ_k is a sequence of arrival times of Poisson process,
- $(V_{1,k}, V_{2,k})$ is a couple of sequence of i.i.d. random variables independent of ε_k and of Γ_k having the same joint distribution of control measure m, which has probability density ϕ
- θ_k is a sequence of i.i.d. random variables that have the uniform distribution on $[-\pi,\pi]$, independent of ε_k, $V_{1,k}$, $V_{2,k}$ and Γ_k.

For the similation ofN_1 and N_2 values of the process Z_{n_1,n_2} where

$(N1, N_2 = 101,501,1001,1501,2001)$, we generate:

- 2000 values of ε_k
- 2000 values of Γ_k
- 2000 values of V_k
- 2000 values of θ_k

and calculate for all $0 \leq n_1 \leq N_1$ and $0 \leq n_2 \leq N_2$

$$Z_{n_1,n_2} = C_\alpha(\int \phi(x_1,x_2)dx_1dx_2)^{1\alpha} \sum_{k=1}^{2000} \varepsilon_k \Gamma_k^{-1\alpha} e^{i(n_1V_{1,k}+n_2V_{2,k})} e^{i\theta_k}$$

The spectral density is taken as $\phi(x_1,x_2) = |x_1x_2|^\beta e^{-|x_1|-|x_2|}$ for $x_1,x_2 \in [-\pi,\pi]$ and $\phi(x_1,x_2) = 0$ otherwise and $\alpha = 1{,}68$ and $k = 4$. The value of β is taken so as to have two cases: β greater than $2k\alpha - 1$ and β less than $2k\alpha - 1$. Afterwards, we generate :

$X_{n_1,n_2} = a + Z_{n_1,n_2}$ where a is chosen equal to 45.

We calculate the estimator $\hat{a}$ given in (1) for different sizes of sample $N_1, N_2 = 101{,}501{,}1001{,}1501{,}2001$. The result is given in the following table:

$\boldsymbol{2k\alpha - 1 = 12.44}$	$\boldsymbol{\beta = 0.5}$	$\boldsymbol{\beta = 30}$
N1= N2=101	$\hat{a} = 30.4$	$\hat{a}$ =37
N1= N2=501	$\hat{a} = 34.7$	$\hat{a}$ =48
N1= N2=1001	$\hat{a} = 50.2$	$\hat{a}$ =42.2
N1= N2=1501	$\hat{a}$ =39.9	$\hat{a}$ =47.1
N1= N2=1201	$\hat{a}$ =43.8	$\hat{a}$ =44.2

We find that the estimator converges more and more to a (a=45) when the size of the sample increases. The approximation is more accurate when β is larger than $2k\alpha - 1$compared to the case where β is smaller than $2k\alpha - 1$.

4. CONCLUSIONS

This work presents an estimate of the constant error, which is added to the observation of the signal (SαS) when the spectral measurement is the sum of a continuous part and a few jumps (mixed measurement). The paper shows that the absence of the energy of signal at frequencies close to zero allows rapid convergence of the estimator. In other words, the energy at frequencies close to zero slows down and disturbs the convergence of our estimator.

This work may have several applications where processes have high variability and are disturbed by constant noise. For instance:

- Dynamic images of an agricultural field taken by drones, to identify weeds. These images can have a great variance, in particular when the images are disturbed by climatic conditions.

- The amount of certain microorganisms in the soil varies in a significant way that can be modelled by alpha stable random field. The measurement of microorganisms can be taken with a constant error and with some jumps when encountering rocks in the ground.

The proposed estimator depends on a smoothing parameter. Among the perspectives of this work is to find the optimal value of this parameter having the best convergence.

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REFERENCES

[1] S. Cambanis, “Complex Symmetric stable variables and processes”, In P.K.SEN, ed, Contributions to Statistics: Essays in Honour of Norman L. Johnson North-Holland. New York,(P. K. Sen. ed.), pp. 63-79 , 1983.
[2] M. Fontaine, K. Sekiguchii, A. Nugraha, A. Aditya, “Alpha-Stable Autoregressive Fast Multichannel Nonnegative Matrix Factorization for Joint Speech Enhancement and Dereverberation”, In Interspeech,. p. 661-665, 2021.
[3] Jung, Paul, et al. "Alpha-Stable convergence of heavy-tailed infinitely-wide neural networks." arXiv preprint arXiv:2106.11064 2021.
[4] E. Masry, S. Cambanis, “Spectral density estimation for stationary stable processes”, Stochastic processes and their applications, 18, pp. 1-31 North-Holland, 1984.

[5] G. Samorodnitsky and M. Taqqu, “Stable non gaussian processes”. Chapman and Hall, New York, 1994.

[6] K., Panki and S. Remning, “Stable process with singular drift”, Stochastic Process. Appl., 124, no. 7, pp. 2479-2516, 2014.

[7] S. Legalive, U. Şimsekli,, A. Liutkus, “Speech enhancement with variational autoencoders and alpha-stable distributions”, In : ICASSP 2019-2019 IEEE International Conference on Acoustics, Speech and Signal Processing (ICASSP). IEEE, p. 541-545. 2019.

[8] C. Zhen-Qing and W. Longmin, “Uniqueness of stable processes with drift”, Proc. Amer. Math. Soc., 144, pp. 2661-2675, 2016.

[9] R. Sabre, “Mixed Spectra for Stable Signals From Discrete Observations”, Signal & Image Processing: An International Journal (SIPIJ) Vol.12, No.5, pp. 21-44, 2021.

[10] M. Liu, Y. Han, Y. Chen, H. Song, Z. Yang and F. Gong, "Modulation Parameter Estimation of LFM Interference for Direct Sequence Spread Spectrum Communication System in Alpha-Stable Noise," in IEEE Systems Journal, vol. 15, no. 1, pp. 881-892, 2021.

[11] R. Sabre, “Estimation de la densité de la mesure spectrale mixte pour un processus symétrique stable strictement stationnaire”, C. R. Acad. Sci. Paris, 319, série I, pp. 1307-1310, 1994.

[12] R. Sabre, , W. Horrigue, J. C. Simon “Discrete Estimation of Spectral Density for Alpha Stable Signals Observed with an Additive Error”, International Journal of Electrical and Computer Engineering, Vo.:14, No.1, pp. 1-5, 2020.

[13] M.H. Bibalan H. Amindavar, M. Amirmazaghani, “Characteristic function based parameter estimation of skewed alpha-stable distribution: An analytical approach”, Signal Processing Vol. 130, pp. 323-336, 2017.

[14] C. Gogineni, S. P. Talebi, S. Werner and D. P. Mandic, "Fractional-Order Correntropy Adaptive Filters for Distributed Processing of α-Stable Signals," in IEEE Signal Processing Letters, vol. 27, pp. 1884-1888, 2020.

[15] X. Yan, G. Liu, H. -C. Wu, G. Zhang, Q. Wang and Y. Wu, "Robust Modulation Classification Over α-Stable Noise Using Graph-Based Fractional Lower-Order Cyclic Spectrum Analysis," in IEEE Transactions on Vehicular Technology, vol. 69, no. 3, pp. 2836-2849, 2020.

[16] C.L. Nikias and M. Shao, “Signal Processing with Alpha-Stable Distributions and Applications”, Wiley, New York, 1995.

[17] N. Azzaoui, L. Clavier, R. Sabre, “Path delay model based on stable distribution for the 60GHz indoor channel”, IEEE GLOBECOM, IEEE, pp. 441-467, 2002.

[18] J.P. Monillet and Yu. Kegen, “Modeling Geodetic Processes with Levy alpha-Stable Distribution and FARIMA”, Mathematical Geosciences. 47, no. 6, pp. 627-646, 2015.

[19] I. Wahyuni, R. Sabre, “Multifocous Image Fusion Using Laplacian Pyramid Technique Based on Alpha Stable Filter” CRPASE Vol. 05(02), 58-62, 2019.

[20] M. Pereyra and H. Batalia, “Modeling Ultrasound Echoes in Skin Tissues Using Symmetric alpha-Stable Processes”, IEEE Transactions on Ultrasonics, Ferroelectrics, and Fr 60 equency Control, 59, no. 1, pp. 60-72, 2012.

[21] Wu. Ligang and W. Zidong, “ Filtering and Control for Classes of Two-Dimensional Systems”, The series Studies in Systems of, Decision and Control, 18 pp. 1-29, 2015.

[22] R. Sabre, “Wavelet Decomposition and Alpha Stable Fusion”, Signal & Image Processing: An International Journal (SIPIJ), Vol.11, No.1, pp. 11-24, 2020.

[23] X. Zhong and A.B. Premkumar, “Particle filtering for acoustic source tracking in impulsive noise with alpha-stable process”, IEEE Sensors Journal, 13, no. 2, pp. 589 – 600, 2012.

[24] K. Panki, K. Takumagai and W. Jiang, “Laws of the iterated logarithm for symmetric jump processes”, Bernoulli, 23, no. 4A, pp. 2330-2379, 2017.

[25] Gao, Mang, Gang Yu, and Tao Wang. "Impulsive gear fault diagnosis using adaptive Morlet wavelet filter based on alpha-stable distribution and kurtogram." IEEE Access 7 (2019): 72283-72296.

[26] R. Sabre, “Spectral density estimate for stationary symmetric stable random field”, Applicationes Mathematcaes, 23, no. 2, pp. 107-133, 1995.

[27] R. Sabre, “Spectral density estimate for alpha-stable p-adic processes”, Revisita Statistica , 72 ,no. 4, pp. 432-448, 2012.

[28] R. Sabre, “Estimation of Additive Error in Mixed Spectra for Stable Precesses”, Revisita Statistica , LXXVII, n.2, pp. 75-90, 2017.

[29] N. Demesh, “Application of the polynomial kernels to the estimation of the spectra of discrete stable stationary processes”. (Russian) Akad.Nauk.ukrain. S.S.R. Inst.Mat. Preprint 64, pp. 12-36, 1988.

[30] A. Janicki and A. Weron, “Simulation and Chaotic Behavior of Alpha-stable Stochastic Processes”. Series: Chapman and Hall/CRC Pure and Applied Mathematics, New York, 1993.

AUTHOR

Rachid SABRE

Received the PhD degree in statistics from the statistics from the University of Rouen, Rouen, France, in 1993 and Habilitation (HdR) from the University of Burgundy, Dijon, France, in 2003.He joined Agrosup Dijon, Dijon, France, in 1995, where he is an Associate Professor. From 1998 through 2010, he served as a member of Institut de Mathématiques de Bourgogne, France. He was a member of the Scientific Council AgroSup Dijon from 2009 to 2013. In 2012, he has been a member of Laboratoire Electronique, Informatique, et Image (Le2i), France. Since 2019 has been a member of Laboratory Biogeosciences UMR CNRS, University of Burgundy. He is author/co-author of numerous papers in scientific and technical journals and conference proceedings. His research interests lie in areas of statistical process and spectral analysis for signal and image processing.