

# THE ECONOMIC PRODUCTIVITY OF WATER IN AGRICULTURE BASED ON ORDERED WEIGHTED AVERAGE OPERATORS

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## **ABSTRACT**

*Water productivity is one of the main indicators used in agriculture. Price of water change from some regions where the price is free to other with a very high price. When water productivity is measured in Euros, to make comparable the results of the regions where the price is free, we need to obtain a correct measurement, which will require setting a market price for water in areas where no price has yet been set. Therefore, the aim of this paper is to propose new productivity indicators based on fuzzy logic, whereby experts' opinions about the possible price of the use of water as well as the annual variability of agricultural prices can be added. Therefore, the fuzzy willingness to pay (FWTP) and fuzzy willingness to accept (FWTA) methodology will be applied to create an artificial water market. The use of fuzzy logic will allow the uncertainty inherent in the experts' answers to be collected. Ordered Weighted Averaging (OWA) operators and their different extensions will allow different aggregations based on the sentiment or interests reflected by the experts. These same aggregators, applied to the prices of the products at origin, will make it possible to create new indicators of the economic productivity of water. Finally, through an empirical application for a pepper crop in south-eastern Spain we can visualize the importance of the different indicators and their influence on the final results.*

## **KEYWORDS**

*Water Economic Productivity, OWA, Fuzzy Willingness to Pay, Fuzzy Willingness to Accept.*

## **1. INTRODUCTION**

Water is an essential resource for agriculture. However, there are regions where this resource is scarce or practically non-existent, such as the Spanish southeast. Here, the water used comes from other basins or even from desalination plants, which entails a significant increase in costs. This situation contrasts with other regions where water exists in abundance and farmers do not pay for it, or they pay minimal costs for the maintenance of traditional channels. For this reason, it is essential to analyse the economic results of water exploitation [1].

Various indicators measure water productivity. Among these are production per cubic meter consumed or income per cubic meter [2]; but the most representative indicator is the income obtained per euro of water consumption by a farmer. However, it is evident that this information is not always available, so it will be necessary to resort to the use of surveys where the high subjectivity inherent in these estimates must be considered [3].

The valuation of environmental assets has been carried out using the contingent valuation method, travel costs methodology, or hedonic prices [4-5]. However, many of these studies do not include the subjectivity contained in the answers of the respondents, since they often have an interest in distorting prices based on their interests. For this reason, on the one hand, by using fuzzy logic the uncertainty of this process can be included; and on the other, by using different aggregators such as Ordered Weighting Aggregators (OWA) [6] and their different extensions such as heavy OWA (HOWA) [7], induced OWA (IOWA) [8], probability OWA (POWA) [9] or induced POWA (IPOWA) [10], the interests of the respondents can be adequately treated.

From these aggregators, new indicators of the economic productivity of water will be created. Firstly, this allows us to collect information on water prices in areas where there is no market; and secondly, it allows for an adequate aggregation of the prices of agricultural products in order to reduce their annual variability [11].

Therefore, the aim of this paper is to propose a new methodology for estimating the economic productivity of water in agricultural companies. To do this, first, the price of water will be estimated using the willingness to pay or willingness to accept methodology by means of fuzzy logic and different aggregators. Then, a reference price of production will be obtained using these aggregators, which allows the economic productivity of water to be obtained. Finally, an application of the economic productivity of water in a greenhouse pepper plantation will be carried out.

## 2. MATERIAL AND METHODS

Definition 1. An OWA operator [6] of dimension  $n$  is a mapping of  $OWA: R^n \rightarrow R$  with an associated weight vector  $W$  of dimension  $n$  such that  $\sum_{j=1}^n w_j = 1$  and  $w_j \in [0,1]$  according to the following expression in which  $b_j$  is the  $j$ th largest element of the collection  $a_i$

$$OWA(a_1, a_2, \dots, a_n) = \sum_{j=1}^n w_j b_j \quad (1)$$

### 2.1. Water Demand Function

For a region where the price of water for agriculture is null or merely nominal, such as paying for certain minimum infrastructure maintenance costs without energy consumption and with minimum personnel cost, the application of a cost can lead to saving water. For this reason, a group of experts (farmers or managers of agricultural companies) are asked to assess their willingness to pay a series of prices. To do this, a series of prices will be provided  $P = \{P_1, P_2, \dots, P_p\}$  in ascending order, with  $P_i < P_{i'}, i < i'$ . These prices must be valued according to the following linguistic scale in which the membership function is indicated: “totally disagree” (0.00), “strongly disagree” (0.20), “disagree” (0.40), “agree” (0.60), “strongly agree” (0.80) and “totally agree” (1.00).

The expert will be presented with the first price,  $P_1$ . If their assessment is “totally disagree”, the price they would be willing to pay would be  $\$0 \text{ m}^{-3}$ . For any other feedback, price  $P_2$  would be offered, and the process would proceed in a similar way. If the answer is “totally disagree”, the process stops, but if any other evaluation is given, then it continues with the following price, and so on.

From this information, it is possible to obtain the price that the expert would be willing to pay to have the water necessary for irrigation. It is important to bear in mind that this is about determining the price they would be willing to pay for their current water needs. To do this, the following algorithm is used.

Step 1. Determination of willingness to pay from expert  $j$  ( $WTP_j$ ) can be obtained as the sum of the products of the price increases of each phase in relation to the previous one ( $\Delta P_i = P_i - P_{i-1}$ ), and the membership function  $\mu_{ij}$  stated by expert  $j$  for phase  $i$ . It is assumed that  $\Delta P_1 = P_1$ .

(2)

As a result, the prices that each of the  $J$  experts will be willing to pay are obtained  $WTP^1 = \{WTP_1^1, WTP_2^1, \dots, WTP_J^1\}$ .

Step 2. Obtaining the willingness to pay weights. They are assigned according to the different aggregator operators. Each aggregator operator will generate a willingness to pay ( $WTP$ ) price.

Step 3. Water demand function. The demand curve is obtained by combining the sum of the experts' weights, for which we have calculated a  $WTP$  equal to or higher than the price,  $P_i$ , for each price presented to the experts ( $P_i$ ), abscissa axis.

$$\mu^1(P_i) = \sum_{j=1}^J \omega_j / WTP_j \geq P_i \quad (3)$$

## 2.2. Water Supply Function

The water supply function is obtained in a similar way to the water demand function. In this case, the current owners of the water will be asked the price they require in exchange for giving up their rights. The evaluation will start at higher prices, and they will be asked about their willingness to accept each price in exchange for permitting the use of their water. In this case, obtaining willingness to accept (WTA) will be obtained from the maximum price and subtracting the membership function of each lower price by the price reduction that it implies in relation to the previous one.

## 2.3. Equilibrium Price

One group of experts will have expressed their willingness to pay and the other their willingness to accept a price. Membership functions of willingness to pay decrease with  $P$  and willingness to accept increases with  $P$ . Therefore, the objective is to find a price for which both membership functions are equal. To do this, the following algorithm is presented.

Step 1. Starting from the price series of the demand function with their corresponding membership functions  $(P_1, \mu_1^P), (P_2, \mu_2^P), \dots, (P_n, \mu_n^P)$  and from the supply prices  $(P_1, \mu_1^A), (P_2, \mu_2^A), \dots, (P_n, \mu_n^A)$ .

Step 2. Search for the point of intersection, placed between two consecutive prices  $P_s$  y  $P_{s+1}$  such that  $\mu_s^P > \mu_s^A$  and  $\mu_{s+1}^P < \mu_{s+1}^A$ .

Step 3. Obtaining the intersection point by interpolation between the points indicated in step 2.

$$\left. \begin{aligned} \frac{x - \mu_s^P}{y - P_s} &= \frac{\mu_{s+1}^P - \mu_s^P}{P_{s+1} - P_s} \\ \frac{x - \mu_s^A}{y - P_s} &= \frac{\mu_{s+1}^A - \mu_s^A}{P_{s+1} - P_s} \end{aligned} \right\} \quad (4)$$

It is evident that on many occasions, the experts' opinions are biased out of the fear that they may imply a higher cost in the future (water users) or a lower income (water providers). The use of OWA equilibrium prices will allow these circumstances to be properly incorporated.

## 2.4. Water Productivity

Water productivity is measured in different ways, such as productivity in Euros per cubic meter of water consumed or income per cubic meter [12]. From an economic point of view, the income from the cost of water is of more interest; that is to say, the quotient between a farm's income (total production,  $V$ , multiplied by the average sale price  $P^V$ ) and the cost of water (cubic meters consumed,  $W$ , multiplied by its price  $P^W$ ).

$$EPW = \frac{V \times P^V}{W \times P^W} \quad (5)$$

The average sales price of each campaign is different, so to obtain a reference price, it is necessary for an aggregation system to be established.

## 2.5. OWA Operators in Water Economic Productivity

Aggregation only with arithmetic means or weighted means prevents other aspects from being considered. For example, subjective considerations on whether it is convenient for the respondents to obtain high or low results regarding the price of water, which could favour their interests. With respect to product sale price, the problem is that only historical prices are available, so it is necessary to incorporate relevant information not included in said prices [13]. We propose the following operators:

**Definition 2.** An OWA-EPW operator of dimension  $n+m$  is a mapping of  $OWA-EPW: R^{n+m} \rightarrow R$  with an associated weight vector  $W$  of dimension  $n+m$  such that  $\sum_{j=1}^n w_j = 1$ ,  $\sum_{j=n+1}^{n+m} w_j = 1$  and  $w_j \in [0,1]$  according to the expression:

$$OWA-EPW(p_V^1, \dots, p_V^n, p_W^1, \dots, p_W^m) = \frac{OWA(p_V^1, \dots, p_V^n)}{OWA(p_W^1, \dots, p_W^m)} \quad (6)$$

**Definition 3.** A HOWA-EPW operator is a mapping of  $OWA-EPW: R^{n+m} \rightarrow R$  associated with a weighting vector  $W$  of dimension  $n+m$ , such that  $w_j \in [0,1]$ ,  $1 \leq \sum_{j=1}^n w_j \leq n$  and  $1 \leq \sum_{j=n+1}^{n+m} w_j \leq m$  such that:

$$HOWA-EPW(p_V^1, \dots, p_V^n, p_W^1, \dots, p_W^m) = \frac{HOWA(p_V^1, \dots, p_V^n)}{HOWA(p_W^1, \dots, p_W^m)} \quad (7)$$

Definition 4. An IOWA-EPW operator of dimension  $n+m$  is a mapping of  $IOWA-EPW: R^{n+m} \times R^{n+m} \rightarrow R$  that has an associated weighting vector,  $W$  of dimension  $n+m$  where  $\sum_{j=1}^n w_j = 1$ ,  $\sum_{j=n+1}^{n+m} w_j = 1$  and  $w_j \in [0,1]$  such that:

$$IOWA-EPW(\langle u_{p_V}^1, p_V^1 \rangle, \dots, \langle u_{p_V}^n, p_V^n \rangle, \langle u_{p_W}^1, p_W^1 \rangle, \dots, \langle u_{p_W}^m, p_W^m \rangle) = \frac{IOWA(\langle u_{p_V}^1, p_V^1 \rangle, \dots, \langle u_{p_V}^n, p_V^n \rangle)}{IOWA(\langle u_{p_W}^1, p_W^1 \rangle, \dots, \langle u_{p_W}^m, p_W^m \rangle)} \quad (8)$$

Definition 5. A POWA-EPW operator of dimension  $n+m$  is a mapping of  $POWA-EPW: R^{n+m} \rightarrow R$  having an associated weighting vector  $P$ , where  $p_j \in [0,1]$ ,  $\sum_{j=1}^n p_j = 1$  and  $\sum_{j=n+1}^{n+m} p_j = 1$  such that:

$$OWA-EPW(p_V^1, p_V^2, \dots, p_V^n, p_W^1, p_W^2, \dots, p_W^m) = \frac{POWA(p_V^1, p_V^2, \dots, p_V^n)}{POWA(p_W^1, p_W^2, \dots, p_W^m)} \quad (9)$$

Definition 6. An IPOWA-EPW operator of dimension  $n+m$  is a mapping of  $IPOWA: R^{n+m} \times R^{n+m} \rightarrow R$  that has an associated weight vector  $W$  of dimension  $n+m$ , where  $\sum_{j=1}^n w_j = 1$ ,  $\sum_{j=n+1}^{n+m} w_j = 1$  and  $w_j \in [0,1]$  so that:

$$IPOWA-EPW(\langle u_{p_V}^1, p_V^1 \rangle, \dots, \langle u_{p_V}^n, p_V^n \rangle, \langle u_{p_W}^1, p_W^1 \rangle, \dots, \langle u_{p_W}^m, p_W^m \rangle) = \frac{IPOWA(\langle u_{p_V}^1, p_V^1 \rangle, \dots, \langle u_{p_V}^n, p_V^n \rangle)}{IPOWA(\langle u_{p_W}^1, p_W^1 \rangle, \dots, \langle u_{p_W}^m, p_W^m \rangle)} \quad (10)$$

Expressions (6) to (10) can have different combinations. In particular, the proposed definitions can be used or simplified by assuming that the OWA and its extensions are only applied to water prices or agricultural sales prices.

### 3. EMPIRICAL APPLICATION

The aim was to analyse the water productivity of one-hectare plantation of Lamuyo peppers under greenhouse in 2021. Average production was 35,530 kg ha<sup>-1</sup> and water consumption was estimated at 8,200 m<sup>3</sup>. Since the plantation had unlimited access to water at practically zero cost, we wanted to know the economic productivity of the water. To do this, experts were asked for their willingness to pay and to accept.

Table I shows the weighting vectors for the OWAs and HOWAs, the induced variable (number of cultivated hectares), expert number, and the probability assigned to each of the eight that act as requestors and each of the six that act as bidders. The induced variable for bidders was their responsibility in the firm they worked for, ranging from 1 to 10. In both cases,  $\beta = 0.4$  has been considered.

Table I. Weighting vectors, experts (Exp), and probabilities (p).

Demand					Supply				
OWA	HOWA	Induced	Exp	p	OWA	HOWA	Induced	Exp	p
0.25	0.30	15	1	0.02	0.26	10	9	1	0.15
0.20	0.25	10	2	0.25	0.22	8	7	2	0.20
0.18	0.20	8	3	0.10	0.17	6	4	3	0.12
0.15	0.15	7	4	0.20	0.15	4	3	4	0.21
0.08	0.10	4	5	0.18	0.12	3	6	5	0.13
0.07	0.10	6	6	0.12	0.08	2	10	6	0.19
0.05	0.05	5	7	0.08					
0.02	0.05	3	8	0.05					

Table II shows the opinions about each price. Expert 1 (consumer) was not willing to pay any amount, but expert 2 totally agreed up to the price of €0.10 m<sup>-3</sup>, and strongly agreed to the price of €0.15 m<sup>-3</sup>, etc. For the experts who acted as bidders, they started at higher prices, so they first agreed to accept a price of up to €0.15 m<sup>-3</sup>, but for €0.10 m<sup>-3</sup>, they only totally agreed, and for €0.05 m<sup>-3</sup>, they agreed.

Table II. Willingness to Pay (WTP) and Willingness to Accept (WTA)

Demand	Price							
Expert	0.05	0.1	0.15	0.2	0.25	0.3	0.35	WTP
1	0	0	0	0	0	0	0	0.00
2	1	1	0.8	0.6	0.2	0	0	0.18
3	1	0.8	0.8	0.2	0.2	0	0	0.15
4	1	1	1	1	1	1	0.4	0.32
5	1	1	0.8	0.6	0.4	0.2	0	0.20
6	0.8	0.4	0.2	0	0	0	0	0.07
7	1	1	0.4	0	0	0	0	0.12
8	1	1	1	0.8	0.6	0.6	0.2	0.26
Supply								WTA
1	0	0.8	1	1	1	1	1	0.06
2	0	0	0	0	0.2	0.8	1	0.25
3	0	0	0	0	0.2	0.6	0.8	0.27
4	0.4	1	1	1	1	1	1	0.03
5	0	0	0	0	0	0	0.4	0.33
6	0.2	0.4	0.4	0.8	1	1	1	0.11

The demand and supply curves were obtained from these opinions. As can be seen, the equilibrium points (which are analytically shown in Figure 1) differ greatly depending on the aggregators used. The membership function corresponding to the demand aggregator IOWA is systematically lower than the rest, being the highest those corresponding to the OWA and HOWA aggregators. On the contrary, the supply function of the aggregators IPOWA and IOWA present the highest values, being the lowest the HOWA operator. In addition to those described, multiple combinations could be made between an aggregator for demand and another for supply. In general, they range from €0.14 to €0.27 m<sup>-3</sup>.

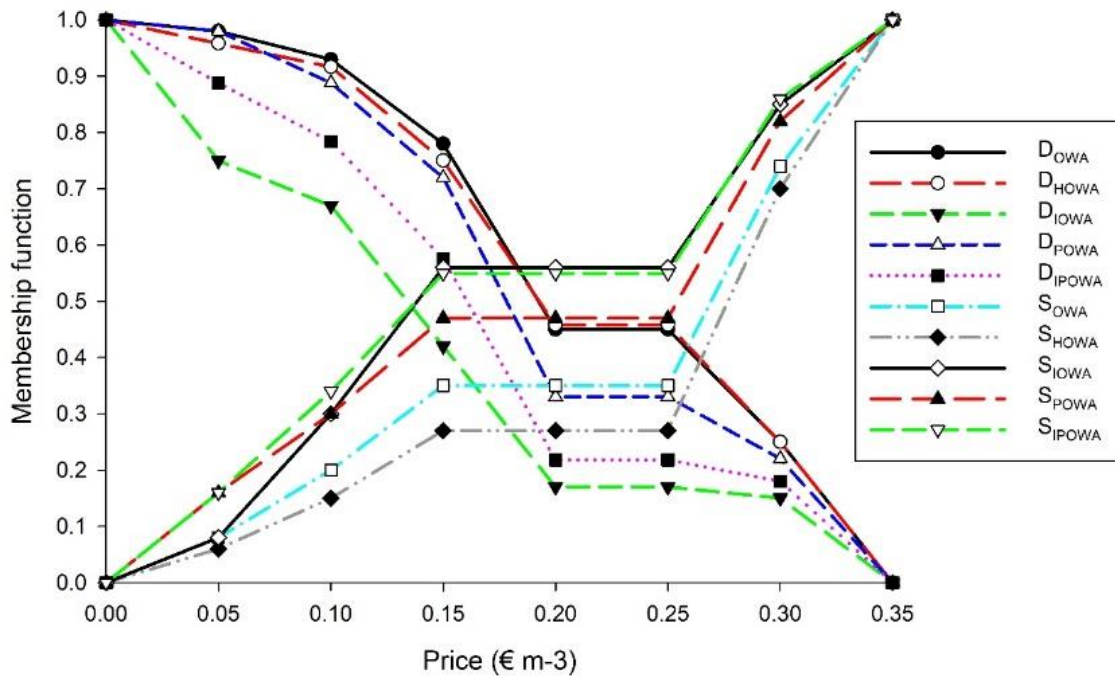


Figure 1. Water demand (D) and supply (S) functions for Ordered Weighting Average (OWA), Heavy OWA (HOWA), Induced OWA (IOWA), Probability OWA (POWA), and Induced POWA (IPOWA).

The historical prices of peppers are a known variable, and they are shown in Table III [14]. Their variability makes the inference of future prices complex. In fact, prices have ranged from €0.56 kg<sup>-1</sup> in 2014 to €1.00 kg<sup>-1</sup> in 2021. Table III shows the weighting vectors for each aggregator and the probabilities assigned to each expert.  $\beta = 0.4$  has been considered.

Table III. Annual Prices of Pepper and Weighting Vectors and Probabilities for Each Expert

Year	Prices	OWA	HOWA	Induced	Expert	Probability
2021	1.00	0.17	20	0.86	1	0.06
2020	0.86	0.15	15	0.87	2	0.14
2019	0.87	0.14	13	0.73	3	0.11
2018	0.73	0.12	12	0.69	4	0.07
2017	0.69	0.11	8	0.89	5	0.08
2016	0.89	0.10	6	0.82	6	0.16
2015	0.82	0.08	5	0.56	7	0.17
2014	0.56	0.06	4	0.78	8	0.08
2013	0.78	0.04	3	0.54	9	0.09
2012	0.54	0.03	3	0.68	10	0.04

Finally, the equilibrium prices of water, the average prices of peppers, and the economic productivity of water are given (Table IV). The results show a wide dispersion, which demonstrates the importance of this study since the value of productivity is highly influenced by the type of aggregator used.

Table IV. Water and Pepper Prices, and Productivity by Aggregators

Operator	Water price (€ m <sup>-3</sup> )	Pepper price (€ kg <sup>-1</sup> )	Productivity
OWA-EPW	0.26	0.77	12.97
HOWA-EPW	0.26	0.84	13.67
IOWA-EPW	0.14	0.79	25.25
POWA-EPW	0.18	0.79	18.77
IPOWA-EPW	0.15	0.81	23.06

The use of different aggregators allows obtaining several values for water and pepper price. In particular, water price ranges from €0.26 m<sup>-3</sup> for OWA-EPW and HOWA-EPW and €0.15 m<sup>-3</sup>, and pepper price from €0.77 to €0.84 kg<sup>-1</sup>. As a result, and according to pepper production and water consumption, productivity ranges from 12.97 to 23.06. The option for one or another aggregator will depend, among others, on the optimism and pessimism degree. Anyway, this methodology generalizes the productivity ratio, even for the case in which the water is free.

#### 4. CONCLUSIONS

The aim of this paper is to introduce a new formulation in the traditional calculation of the economic productivity of water, using OWA-EPW, HOWA-EPW, IOWA-EPW, POWA-EPW, and IPOWA-EPW. The advantage of these operators is that they provide new ways of aggregating prices and expert opinions. This makes it possible to visualise the potential risks of certain crops not becoming profitable because it takes many years at exiguous prices. This form of aggregation allows for simplifications if circumstances change. The results or the empirical application show how productivity can almost double depending on the aggregator used. Results of the empirical application show a range of productivity values from 12.97 to 23.06. The choice for one or another will depend on the optimism or pessimism degree.

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